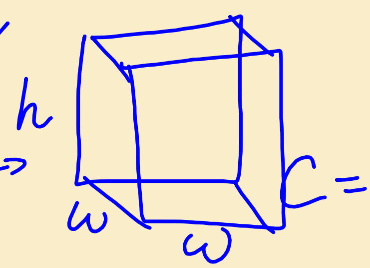
CALCULUS SEMESTER 2 REVIEW

Optimization

$V = 128\ m^3$

$2.$

$0.50 \rightarrow$



$\nearrow \$2$

$C = \$2w^2 + (\$0.5)4wh$

$* \quad C = 2w^2 + 2wh$

$w^2 h = 128$

$h = \frac{128}{w^2}$

$h = \frac{128}{4^2} = 8$

$w: (0, \infty)$

$C = 2w^2 + 2w\left(\frac{128}{w^2}\right)$

$\boxed{4m \times 4m \times 8m}$

$\lim_{w \to 0^+} 2w^2 + \frac{256}{w} = +\infty$

$* \quad C = 2w^2 + \frac{256}{\cancel{w}} w^{-1}$

$\lim_{w \to \infty} 2w^2 + \frac{256}{w} = \infty + 0 = \infty$

$C' = 4w - \frac{256}{w^2}$

$0 = 4w - \frac{256}{w^2}$

$4 \,|\, 32 + 64 = 96$

$\cancel{w^2}\frac{256}{\cancel{w^2}} = 4w \cdot w^2$

$256 = 4w^3$

$\sqrt[3]{64} = \sqrt[3]{w^3}$

$4 = w$

Integration

$$\int \sin x\, dx = -\cos x + C$$
$$\int \cos x\, dx = \sin x + C$$
$$\int \sec^2 x\, dx = \tan x + C$$
$$\int \csc^2 x\, dx = -\cot x + C$$
$$\int \sec x \tan dx = \sec x + C$$
$$\int \csc x \cot x\, dx = -\csc x + C.$$

$$\int e^x dx = e^x + C$$
$$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$$
$$\int \frac{1}{x} dx = \ln|x| + C$$
$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$
$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$
$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

Methods:

1) Power Rule
2) U-Subst.
3) Inv Trig Func.
4) Integ by parts
5) Trig Integrals

$\int \sin 4x \cos 8x\, dx$ – Sum & prod. identity

$\int \sin^3 x \cos^4 x\, dx$ – split off $\sin x$
– use $\sin^2 x + \cos^2 x = 1$
– U-sub

$\int \sin^2 x \cos^4 x\, dx$
– Rewrite as $(\ ^2)^-$
– Use double angle Identities

6) Trig Substitution
7) Partial Fractions

List:

A derivative represents. . .

Integration represents

Give:

$$W = \int \rho \; A(x) \cdot \text{depth} \, dx$$

$$\underset{\text{Force}}{\text{Fluid}} = \int \rho \; l(x) \, h(x) \, dx$$

Know:

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Hyperbolic Functions

$$\frac{d}{dx}\cosh x = +\sinh x$$

$$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$f(x) = \sinh(e^{3x}) \cdot \operatorname{sech}(x^3)$$

Product Rule

$$f'(x) = \underline{\sinh(e^{3x})} \cdot -\underline{\operatorname{sech} x^3 \tanh x^3 \cdot 3x^2} + \underline{\operatorname{sech}(x^3)} \cdot \cosh(e^{3x}) \cdot e^{3x} \cdot 3$$

$$\int x \tanh^5(3x^2)\operatorname{sech}^2(3x^2)\,dx$$

$u = \tanh(3x^2)$

$du = \operatorname{sech}^2(3x^2) \cdot 6x$

$$\int \cancel{x} \cdot u^5 \cancel{\operatorname{sech}^2(3x^2)} \frac{du}{6\cancel{x}\cancel{\operatorname{sech}^2(3x^2)}}$$

$$\frac{1}{6}\int u^5\,du$$

$$\frac{1}{6}\frac{u^6}{6} + C$$

$$\frac{1}{36}\tanh^6(3x^2) + C$$

Power Rule

$$\int\left(\frac{4}{x}+\frac{1}{2x^3}\right)dx$$

$$\int\left(4\cdot\frac{1}{x}+\frac{1}{2}x^{-3}\right)dx$$

$$= 4\ln|x| + \frac{1}{2}\frac{x^{-2}}{-2} + C$$

$$4\ln|x| - \frac{1}{4x^2} + C$$

f) $\displaystyle\int \frac{6}{9+49x^2}\,dx$

$u = \frac{7}{3}x$

$du = \frac{7}{3}dx$

$\frac{3}{7}du = dx$

$$\frac{6}{9}\int \frac{1}{1+\frac{49}{9}x^2}\,dx \qquad \left(\frac{7}{3}x\right)^2$$

$$\frac{2}{\cancel{3}}\int \frac{1}{1+u^2}\,\frac{\cancel{3}}{7}\,du$$

$$\frac{2}{7}\cdot\int\frac{1}{1+u^2}\,du$$

$$\frac{2}{7}\tan^{-1}u + C$$

$$\frac{2}{7}\tan^{-1}\left(\frac{7}{3}x\right) + C$$

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$$\frac{d}{dx}\int_2^x \frac{t^2\sqrt{4t-1}}{\sin^{-1} t^2}\,dt = \frac{x^2\sqrt{4x-1}}{\sin^{-1} x^2}$$

$$\frac{d}{dx}\int_3^{x^3} \frac{4t^2-5}{e^{3t-2}}\,dt = \frac{4(x^3)^2-5}{e^{3x^3-2}}\cdot 3x^2$$

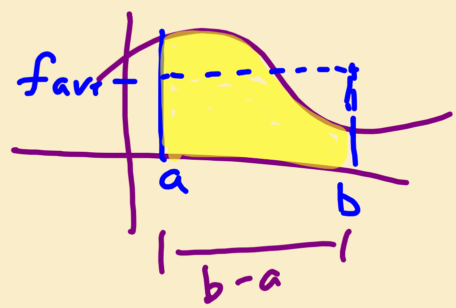
$$= \frac{3x^2(4x^6-5)}{e^{3x^3-2}}$$

10(a)

Find the average value (f_{ave}) of $f(x) = x^3 + 2$ $[0, 3]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{3-0}\int_0^3 (x^3+2)\,dx$$



$$\int x(4+2x)^8\,dx$$

$$\frac{d}{dx}\int_2^{x^3} \frac{(5-t^3)^7}{\sin t}\,dt$$