Semester Review Day 2

Find Area.

$f(x) = 2|x+3| - 4$

from $x = -4$ to $x = 0$

$2(x+3) - 4 = 2x + 2$

$-2(x+3) - 4 = -2x - 10$

$$-\int_{-4}^{-3} (-2x-10)\,dx - \int_{-3}^{-1} (2x+2)\,dx + \int_{-1}^{0} (2x+2)\,dx$$

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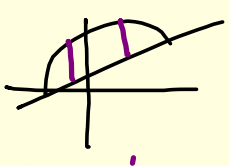
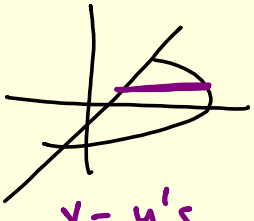
(2) Top-Bottom or R-L



$y = x$'s

$\int_a^b x$'s

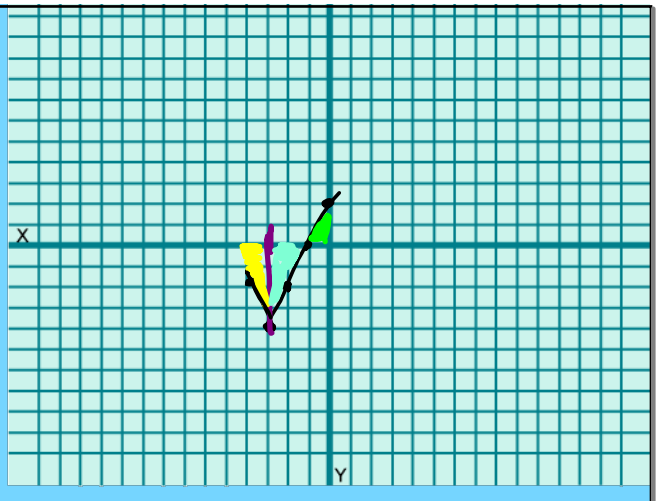
a: x-coord

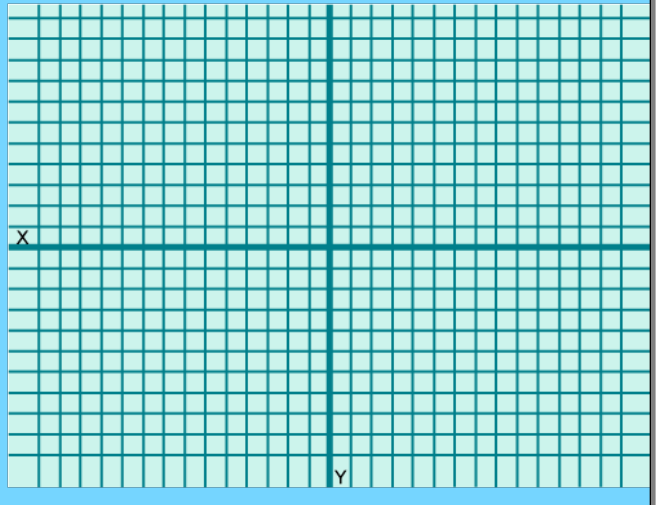


$x = y$'s

$\int_a^b y$'s

a: y's





May 2, 2023

Volume

Disk

$$\pi \int_a^b \left(r_o^2 - r_i^2\right) dx$$

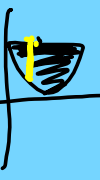
▯ is ⊥ to axis of rev

Shell

$$2\pi \int_a^b r\,(f-g)\,dx$$

▯ is ∥ to axis of rev.

Slicing



isosceles Rt Δ

$A = \frac{1}{2}x^2$

Semicircles $\frac{1}{2}\pi r^2$

Watch out for diam.

If rect is Vertical = Top − Bottom $y = x$'s

If rect is horiz = R − L $x = y$'s

Length of Curve

$$\int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

Surface Area of Rev

$$2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2}\,dx$$

13(b) $y^2 = x^2 - 16$ $y=0$ $x=5$

Disk around y axis

$y = \pm\sqrt{x^2-16}$

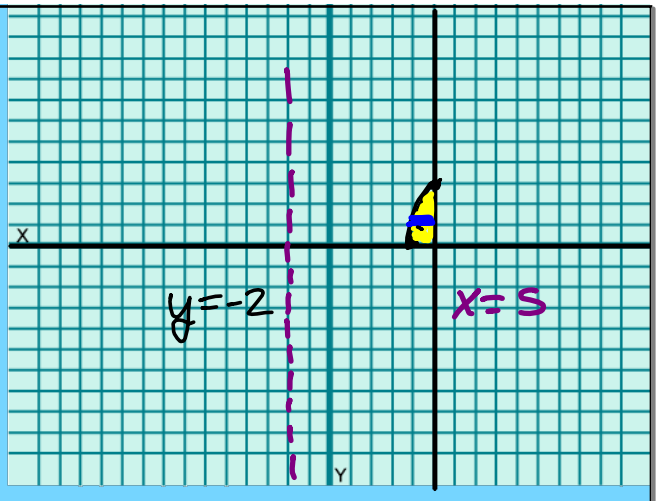
4	0
5	3

$\sqrt{y^2+16} = \sqrt{x^2}$

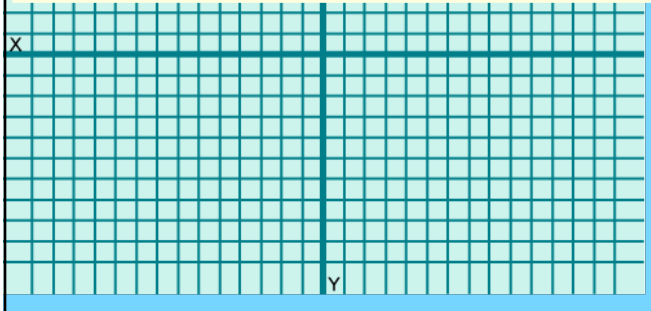
$\sqrt{y^2+16} = x$

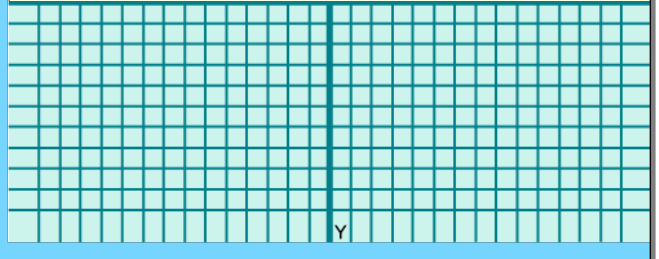
$$\pi \int_a^b \left(r_o^2 - r_i^2\right) dx$$

$$\pi \int_0^3 \left[5^2 - \left(\sqrt{y^2+16}\right)^2\right] dy$$



$$\pi \int_0^3 (5-(-2))^2 - \left(\sqrt{y^2+16}-(-2)\right) dy$$





14(a) $y = x^2$, $y = 0$, $x = 2$

about x-axis

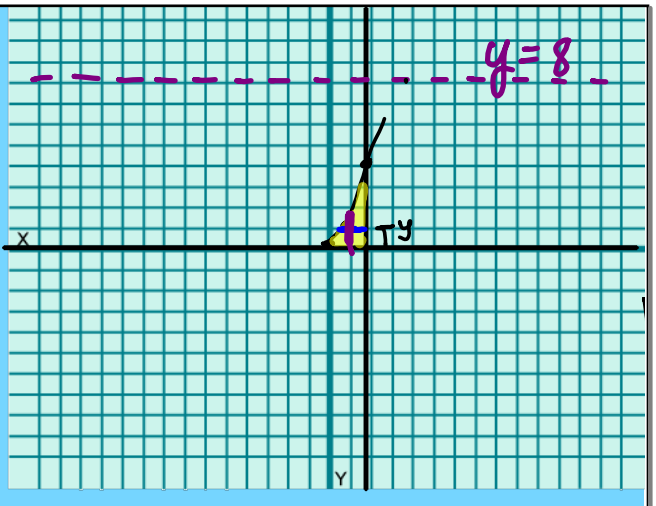
∥ is parallel

$\sqrt{y} = x$

$2\pi \int r\,(f - g)\,dx$

$2\pi \int_0^4 y\,(2 - \sqrt{y})\,dy$

$2\pi \int_0^4 (8 - y)(2 - \sqrt{y})\,dy$

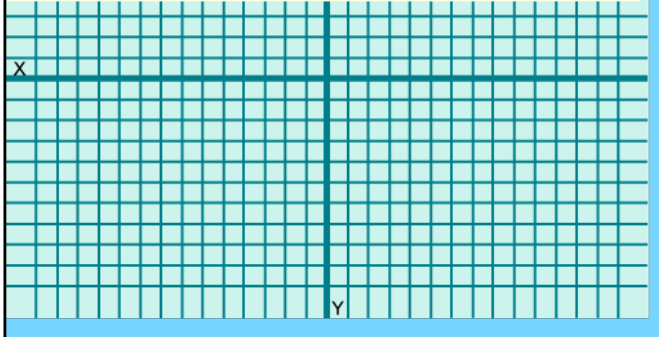


Slicing — ↑ semicircles

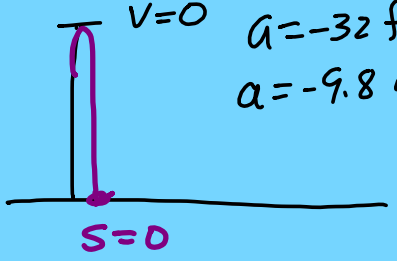
$y = x^2$ $y = 0$

$A = \frac{1}{2}\pi r^2$

$\frac{1}{2}\pi \int_0^2 \left(\frac{x^2 - 0}{2}\right)^2 dx$



Accel, Velocity, Position



$a = -32\ ft/s^2$

$a = -9.8\ m/s^2$

Given $a(t) = -2$

Work $= \int F(x)\,dx$

Springs $F(x) = kx$

$\int 7x$

x ↑ Distance from natural length

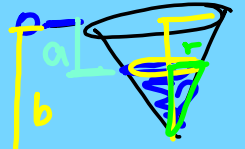
Fluid Force

$\int_0^b \rho\, l(x)\, h(x)\, dx$

62.4 lb/ft^3

9810 N/m^3

Pump Problems



$\int_a^b \rho\, A(x)\, \text{depth}\, dx$