More Derivatives

Product Rule

$$\frac{d}{dx} f\cdot g = f\cdot g' + g\cdot f'$$

$$= \text{1st}\cdot d'\text{2nd} + \text{2nd}\cdot d'\text{1st}$$

$f(x) = 3x^7 \cdot 5x^4$

$\cancel{f'(x) = 21x^6 \cdot 20x^3}$

$\cancel{= 420x^9}$

$f'(x) = 3x^7 \cdot 20x^3 + 5x^4 \cdot 21x^6$

$= 60x^{10} + 105x^{10}$

$= 165x^{10}$

$f(x) = 15x^{11}$

$f'(x) = 165x^{10}$

$f(x) = (7x^5 - 3x^8 - 2)(8x - 4x^9 - 27x^0)$

$$f'(x) = \underset{\text{1st}}{(7x^5 - 3x^8 - 2)}\underset{d'\text{2nd}}{(8 - 36x^8)} + (8x - 4x^9 - 27)(35x^4 - 24x^7)$$

QUOTIENT RULE

$f(x) = \frac{1}{x} = x^{-1-1}$

$f'(x) = -1x^{-2} = \frac{-1}{x^2}$

$$\frac{d}{dx}\frac{f}{g} = \frac{g \cdot f' - f \cdot g'}{g^2}$$

$$= \frac{\text{low} \cdot d'\text{high} - \text{high} \cdot d'\text{low}}{\text{low}^2}$$

$$f(x) = \frac{x^4 - 7x^3 + 8}{2x^5 - 17x^2}$$

$$f'(x) = \frac{\overset{\text{low}}{(2x^5 - 17x^2)} \cdot \overset{d'\text{high}}{(4x^3 - 21x^2)} - \overset{\text{high}}{(x^4 - 7x^3 + 8)}\overset{d'\text{low}}{(10x^4 - 34x)}}{(2x^5 - 17x^2)^2}$$

CHAIN RULE

$$\frac{d}{dx} f\left[g\left(h(x)\right)\right] = f'\left[g(h(x))\right] \cdot g'(h(x)) \cdot h'(x)$$

Use when you have a quantity raised to a power

$$f(x) = \left(4x^3 - 7x^2 + 4\right)^8$$

$$f'(x) = 8\left(4x^3 - 7x^2 + 4\right)^7 \cdot \left(12x^2 - 14x\right)$$

$$f(x) = \sqrt[2]{x^2 + 3x - 5(x^2+4)^9} = \left(x^2 + 3x - 5(x^2+4)^9\right)^{1/2}$$

$$f'(x) = \frac{1}{2}\left(x^2 + 3x - 5(x^2+4)^9\right)^{-1/2} \cdot \left[2x + 3 - 45(x^2+4)^8 \cdot 2x\right]$$

$$f(x)=\frac{(x^5-4x^3+8x)(7x^2-3x+10)}{(x^9-3)^8}$$

$$f'(x)=\frac{\overset{\text{low}}{(x^9-3)^8}\cdot\overbrace{\left[\overset{\text{1st}}{(x^5-4x^3+8x)}\overset{\text{d'2nd}}{(14x-3)}+\overset{\text{2nd}}{(7x^2-3x+10)}\overset{\text{d'1st}}{(5x^4-12x^2+8)}\right]}^{\text{d'high}}-\overbrace{(x^5-4x^3+8x)(7x^2-3x+10)}^{\text{high}}\cdot\overbrace{8(x^9-3)^7\cdot 9x^8}^{\text{d'low}}}{\left[(x^9-3)^8\right]^2}$$