ABSOLUTE VALUE INEQUALITIES

$|-5| = 5$

$|x| = 4$

$x = 4$ or $x = -4$

$|x+6| = 9$

-35

$x+6 = 9$ $\quad$ $x+6 = -9$

$x = 3$ $\quad$ $x = -15$

$-2|x+2| + 12 = 0$

$\quad -12 \quad -12$

$\frac{-2|x+2|}{-2} = \frac{-12}{-2}$

$|x+2| = 6$

$x+2 = 6$ $\quad$ $x+2 = -6$

$x = 4$ $\quad$ $x = -8$

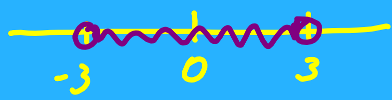
1) Isolate the abs value
2) Write + solve 2 equations

$\frac{-5|3x+1|}{-5} = \frac{20}{-5}$

$|3x+1| = -4$

No solution

$|x| < 3$



→ Less ThAND

$|x| \geq 2$

→ GREAT OR

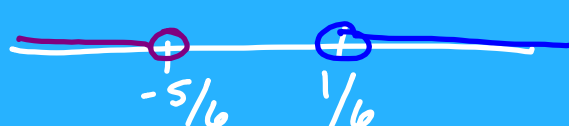
Abs value must be isolated

$$4|6x+2| + 20 > 32$$
$$\quad -20 \quad\quad -20$$

$$\frac{4|6x+2|}{4} > \frac{12}{4}$$

$$|6x+2| > 3$$

$6x+2 > 3$	OR	$6x+2 < -3$ (switch)
$-2 \quad -2$		$-2 \quad -2$
$6x > 1$		$\frac{6x}{6} < \frac{-5}{6}$
$x > 1/6$		$x < -5/6$



$$x < -5/6 \text{ OR } x > \frac{1}{6}$$

$|2x+3| = -6$
No sol.

$|2x+3| < -6$
No sol

$|2x+3| > -6$
$\mathbb{R}$

$-5|6x-8| + 45 > -15$
$\quad\quad\quad\quad -45 \quad -45$

$\frac{-5|6x-8|}{-5} > \frac{-60}{-5}$ switch!

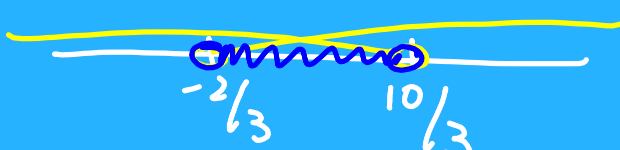
$|6x-8| < 12$

$6x-8 < 12$ AND $6x-8 > -12$ (switch both)

$6x < 20 \qquad 6x - 8 + 8 > -12 + 8$

$\frac{6x}{6} < \frac{20}{6} \qquad \frac{6x}{6} > \frac{-4}{6}$

$x < 10/3 \qquad x > -2/3$



$-2/3 < x < 10/3$

MATRIX ARITHMETIC

$$\begin{bmatrix} 5 & 6 \\ -2 & 3 \\ 4 & 5 \end{bmatrix} \qquad \begin{bmatrix} -2 & 3 & 4 & 8 & 9 \\ 7 & 2 & -1 & 0 & 4 \end{bmatrix}$$

3 x 2   2 x 5

Dimensions:

\# of rows x # of columns

Matrix – a rectangular array of numbers enclosed in brackets

$$3\begin{bmatrix} 2 & 6 \\ -1 & 5 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 0 & 8 \\ -5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 18 \\ -3 & 15 \\ 12 & 9 \\ \cancel{8} & \cancel{2} \end{bmatrix} + \begin{bmatrix} -4 & +1 \\ 0 & -8 \\ +5 & -6 \end{bmatrix} = \begin{bmatrix} 2 & 19 \\ -3 & 7 \\ 17 & 3 \end{bmatrix}$$

Addition/Subtraction = must have same dimensions.

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$$\begin{bmatrix} 3 & -2 & 4 \\ 1 & 0 & -5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ -2 & 6 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 15+4+-4 & 0+-12+12 \\ & \end{bmatrix} = \begin{bmatrix} 15 & 0 \\ & \end{bmatrix}$$

$$2 \times 3 \quad 3 \times 2 \quad = \quad 2 \times 2$$

must be same

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$$\frac{5x^2-6x+12}{x^2(x^2+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+3}$$

$$5x^2-6x+ \;=\; Ax(x^2+3) + B(x^2+3) + x^2(Cx+D)$$

Trig Review

$$\cos\frac{4\pi}{3} - \cot^2\left(-\frac{5\pi}{6}\right)$$

$1\frac{1}{3}$

$$\sin\frac{3\pi}{2}\ \sec\frac{15\pi}{4}$$

$3\frac{3}{4}$

sin θ, csc θ	All
tan θ, cot θ	cos θ, sec θ

0, π/6, π/4, π/3, π/2

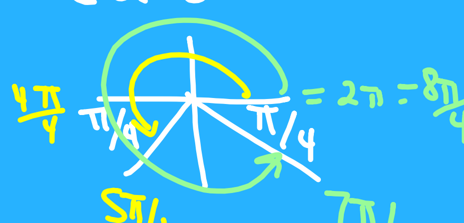
$$\frac{-\frac{1}{2} - (\sqrt{3})^2}{(-1)(+\sqrt{2})} = \frac{-\frac{1}{2} - 3}{-\sqrt{2}} = \frac{+\frac{7}{2}\cdot\sqrt{2}}{+\sqrt{2}\cdot\sqrt{2}} = \frac{\frac{7\sqrt{2}}{2}\cdot\frac{1}{2}}{2} = \frac{7\sqrt{2}}{4}$$

Find θ if $0 \le \theta < 2\pi$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$\frac{5\pi}{6}, \frac{7\pi}{6}$

$$\csc\theta = -\sqrt{2}$$



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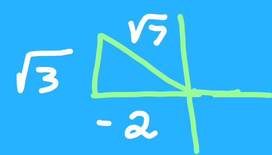
$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

Find $\cot\theta$ given

$\sec\theta = -\frac{\sqrt{7}}{2} \; \frac{r}{x}$ & $\sin\theta > 0$.



$4 + y^2 = 7$

$\sqrt{y^2} = \sqrt{3}$

$\cot\theta = \frac{x}{y} = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

Inverse Trig Functions

$y = \sin\theta$

$\theta = \sin^{-1} y$

$-\{\cos^{-1}x,\ \sec^{-1}x,\ \cot^{-1}x$	All $+$
	$\csc^{-1}x,\ \sin^{-1}x,\ \tan^{-1}x\}\ -$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

$\csc\left(\sec^{-1}\frac{11}{4}\right)$ $\frac{r}{x}$

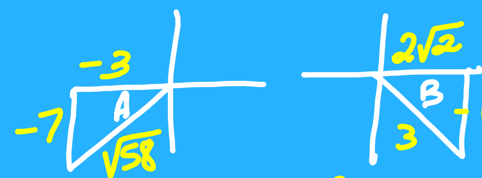
$16 + y^2 = 121$

$\sqrt{y^2} = \sqrt{105}$

$\csc\theta = \frac{r}{y} = \frac{11\cdot\sqrt{105}}{\sqrt{105}\cdot\sqrt{105}} = \frac{11\sqrt{105}}{105}$

If $\tan A = \frac{7}{3}\frac{y}{x}$ & $\csc B = \frac{-3}{1}\frac{r}{y}$ where $\pi < A < \frac{3\pi}{2}$, $\frac{3\pi}{2} < B < 2\pi$

find $\cos(A-B)$.



$\cos A \cos B + \sin A \sin B$

$\left(\frac{-3}{\sqrt{58}}\right)\left(\frac{2\sqrt{2}}{3}\right) + \left(\frac{-7}{\sqrt{58}}\right)\left(\frac{-1}{3}\right)$

$49 + 9 = r^2$

$58 = r^2$

$x^2 + 1 = 9$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm 2\sqrt{2}$

$\frac{-6\sqrt{2} + 7}{3\sqrt{58}} \cdot \frac{\sqrt{58}}{\sqrt{58}} = \frac{-6\sqrt{116} + 7\sqrt{58}}{174} = \frac{-12\sqrt{29} + 7\sqrt{58}}{174}$

($116 = 4 \cdot 29$)

FUNDAMENTAL IDENTITIES

1) $\csc\theta = \frac{1}{\sin\theta}$
2) $\sec\theta = \frac{1}{\cos\theta}$
3) $\cot\theta = \frac{1}{\tan\theta}$
4) $\tan\theta = \frac{\sin\theta}{\cos\theta}$
5) $\cot\theta = \frac{\cos\theta}{\sin\theta}$
6) $\sin^2\theta + \cos^2\theta = 1$
7) $1 + \tan^2\theta = \sec^2\theta$
8) $1 + \cot^2\theta = \csc^2\theta$

$$\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta$$

$1 - 2\sin^2\theta$
$2\cos^2\theta - 1$
$\cos^2\theta - \sin^2\theta$

$$\frac{2\sin\theta\cos\theta}{\sin\theta} - \frac{2\cos^2\theta - 1}{\cos\theta} = \frac{1}{\cos\theta}$$

$$\frac{\cos\theta}{\cos\theta}\cdot\frac{2\cos\theta}{1} - \frac{2\cos^2\theta - 1}{\cos\theta} = \frac{1}{\cos\theta}$$

$$\frac{2\cos^2\theta - 2\cos^2\theta + 1}{\cos\theta}$$

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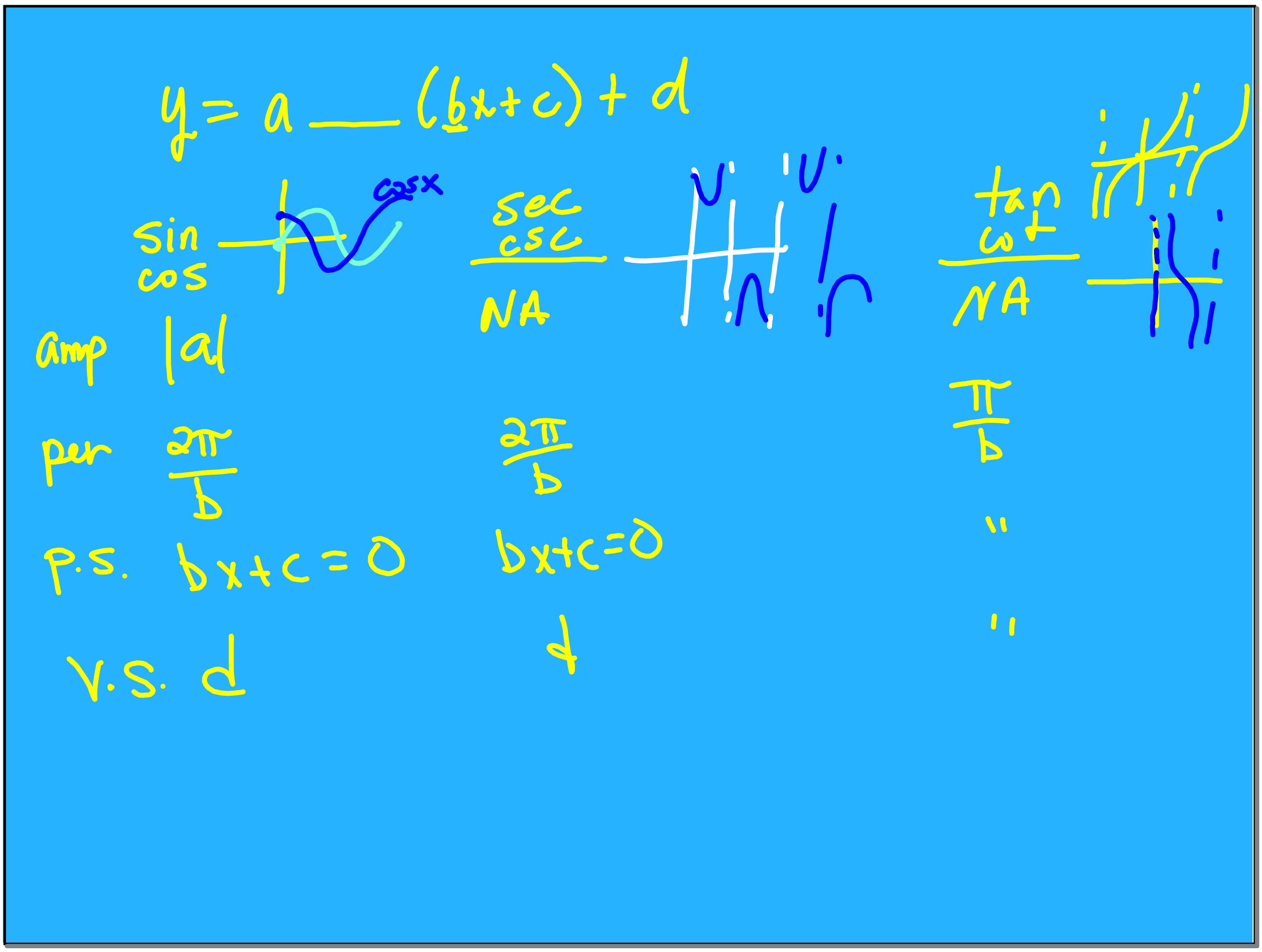

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Law of Sines ASA AAS SSA*

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Law of Cosines SAS SSS

$$c^2 = a^2 + b^2 - 2ab\cos C$$