

# 3-VARIABLE ELIMINATION + CRAMER'S RULE

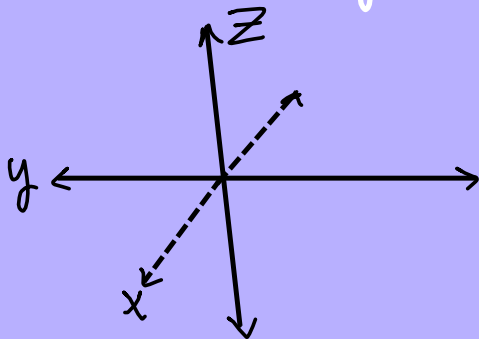
$$\begin{cases} -3x + 8y + 2z = -12 \\ 3x + y + 7z = 38 \\ 4x - 3y + 6z = 47 \end{cases}$$

$$\begin{array}{r} \textcircled{1} \quad -3x - 24y - 6z = 36 \\ \quad + 3x + y + 7z = 38 \\ \hline \quad \quad -23y + z = 74 \end{array}$$

$$\begin{array}{r} \textcircled{3} \quad 2[-23y + z = 74] \\ \quad \quad -35y - 2z = 95 \end{array}$$

$$\begin{array}{r} -46y + 2z = 148 \\ -35y - 2z = 95 \\ \hline -81y = 243 \\ \frac{-81y}{-81} = \frac{243}{-81} \end{array}$$

$$y = -3$$



$$\begin{array}{r} \textcircled{2} \quad -4x - 32y - 8z = 48 \\ \quad + 4x - 3y + 6z = 47 \\ \hline \quad \quad -35y - 2z = 95 \end{array}$$

$$\begin{array}{r} \textcircled{4} \quad -23(-3) + z = 74 \\ \quad \quad 69 + z = 74 \\ \quad \quad \quad -69 \end{array}$$

$$z = 5$$

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$$\begin{array}{r} x + 8(-3) + 2(5) = -12 \\ x - 24 + 10 = -12 \\ x - 14 = -12 \\ \quad \quad +14 \quad \quad +14 \end{array}$$

$$x = 2$$

$$(2, -3, 5)$$

Where 3 planes intersect

## Elimination

- 1) Group 2 equations + eliminate a variable.
- 2) Group a different pair of equations + eliminate the SAME variable.
- 3) Group the 2 resulting equations (with 2 variables) + eliminate a variable.
- 4) Sub answer in a 2-variable equation to find a 2nd variable.
- 5) Sub both answers in a 3-variable equation to get 3rd variable.

# CRAMER'S RULE

$$4x + 2y - z = 15$$

$$2x - y + 5z = 9$$

$$3x + 2y - z = 12$$

$$z = \frac{\begin{vmatrix} \textcircled{4} & \textcircled{2} & \textcircled{15} \\ 2 & -1 & 9 \\ 3 & 2 & 12 \end{vmatrix}}{\begin{vmatrix} \textcircled{4} & \textcircled{2} & -1 \\ 2 & -1 & 5 \\ 3 & 2 & -1 \end{vmatrix}} = \frac{-120 + 6 + 105}{-36 + 34 - 7} = \frac{-9}{-9} = 1$$

$$4 \begin{vmatrix} -1 & 9 \\ 2 & 12 \end{vmatrix} - 2 \begin{vmatrix} 2 & 9 \\ 3 & 12 \end{vmatrix} + 15 \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$$

$$4 \begin{pmatrix} -12 & -18 \\ -30 & 18 \end{pmatrix} - 2 \begin{pmatrix} 24 & -27 \\ -3 & 7 \end{pmatrix} + 15 \begin{pmatrix} 4 & -3 \\ 7 & 7 \end{pmatrix}$$

$$-120 + 6 + 105 = -9$$

$$4 \begin{vmatrix} -1 & 5 \\ 2 & -1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} + -1 \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$$

$$4 \begin{pmatrix} 1 & -10 \\ -9 & 11 \end{pmatrix} - 2 \begin{pmatrix} -2 & -5 \\ -11 & -3 \end{pmatrix} + -1 \begin{pmatrix} 4 & -3 \\ 7 & 7 \end{pmatrix}$$

$$-36 + 34 - 7 = -9$$

$$z = \frac{-9}{-9} = 1$$

## By Calculator

$$\frac{\det \begin{pmatrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{pmatrix}}{\det \begin{pmatrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{pmatrix}}$$

Determinant on Calculator:

Menu - 7 - 3

#18. Look at picture:

$$\text{1st equation: } x + y = 1300$$

$$\text{2nd equation: } x + 2z = 1400$$

$x$  = # of sofas

$y$  = # of love seats

$z$  = # of chairs

You figure out 3rd equation. Then set up Cramer's Rule to solve for all 3 variables.

Use your calculator to work out the determinants. Enter missing variables as 0.