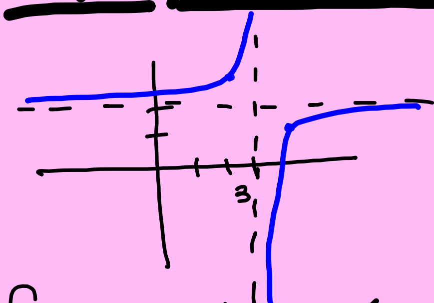
September 12, 2023

ASYMPTOTES & CONTINUITY



$$f(x) = \frac{-1}{x-3} + \frac{2}{1}\frac{(x-3)}{(x-3)}$$

$$= \frac{-1+2x-6}{x-3}$$

$$= \frac{2x-7}{x-3}$$

$$\lim_{x\to\infty} \frac{2x-7}{x-3} = \lim_{x\to\infty} \frac{2\cancel{x}}{\cancel{x}} = 2$$

$$y = 2$$

Vertical

$$\lim_{x\to \#} f(x) = \pm\infty$$

where denom = 0

Horizontal

$$\lim_{x\to\pm\infty} f(x) = \#$$

$$f(x) = \frac{x^2-2x}{x^2+x-6}$$

$(x+3)(x-2)$

$x=-3 \quad x=2$

Vertical

$$\lim_{x\to 2} \frac{x^2-2x}{(x+3)(x-2)} = \frac{0}{0}$$

$$\lim_{x\to 2} \frac{x\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \frac{2}{5}$$

Hole at $x=2$

$$\lim_{x\to -3} \frac{x}{x+3} = \frac{-3}{0}$$

$$\lim_{x\to -3^-} \frac{x}{x+3} = \frac{-}{-} = +\infty$$

$\boxed{\text{Vertical: } x=-3}$

Horizontal

$$\lim_{x\to +\infty} \frac{x}{x} = 1$$

OR

$$\lim_{x\to +\infty} \frac{x^2}{x^2} = 1$$

$\boxed{\text{Horiz. asymp @ } y=1}$

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$$f(x) = \frac{\sqrt{36x^2+11}}{3x-5}$$

Vertical $\boxed{x = 5/3}$

$$\lim_{x \to \frac{5}{3}} \frac{\sqrt{36x^2+11}}{3x-5} = \frac{\#}{0}$$

$$\lim_{x \to 5/3^-} \frac{\sqrt{36x^2+11}}{3x-5} = \frac{+}{-} = -\infty$$

1.67 4.8

Horizontal $\boxed{\begin{matrix} y=2 \\ y=-2 \end{matrix}}$

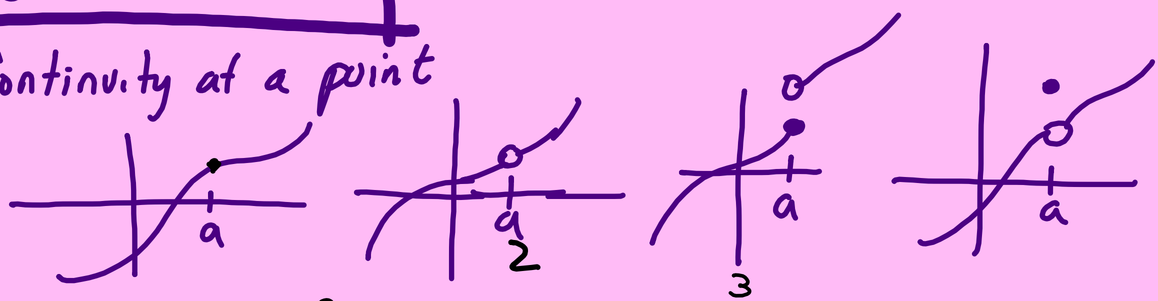
$$\lim_{x \to \infty} \frac{\sqrt{36x^2+11}}{3x-5} = \lim_{x \to \infty} \frac{\sqrt{36x^2}}{3x}$$

$$= \lim_{x \to +\infty} \frac{6|x|}{3x} = \lim_{x \to \infty} \frac{6\cancel{x}}{3\cancel{x}} = 2$$

$$\lim_{x \to -\infty} \frac{6|x|}{3x} = \lim_{x \to -\infty} \frac{-6x}{3x} = -2$$

CONTINUITY – smooth + unbroken

Continuity at a point



1) $f(a)$ is defined.
2) $\lim\limits_{x \to a} f(x)$ exists.
3) $f(a) = \lim\limits_{x \to a} f(x)$

$$f(x) = \begin{cases} 3x+2 & x<1 \\ 7-2x^2 & x \geq 1 \end{cases} ; \; a=1$$

1) $f(1) = 7-2(1)^2 = 5$
2) $\lim\limits_{x \to 1^-} 3x+2 = 5$

   $\lim\limits_{x \to 1^+} 7-2x^2 = 5$

   $\lim\limits_{x \to 1} f(x) = 5$
3) $f(1) = \lim\limits_{x \to 1} f(x)$

   yes, f is continuous.

$$f(x) = \begin{cases} 3x+8 & x<-3 \\ 4 & x=-3 \\ x^2-10 & x>-3 \end{cases} \qquad a=-3$$

$$f(x) = \frac{x+2}{x-4} \quad x \neq 4$$

1) $f(-3) = 4$

2) $\lim\limits_{x \to -3^-} 3x+8 = -1$

$\lim\limits_{x \to -3^+} x^2-10 = -1$

$\lim\limits_{x \to -3} f(x) = -1$

3) $f(-3) \neq \lim\limits_{x \to -3} f(x)$

not continuous

$$f(x) = \sqrt{x^2-4x-21}, \quad (x-7)(x+3)$$

$+ \;]\; - \;[\; +$

$-3 \quad 0 \quad 7$

$(-\infty, -3]$ C

$[7, \infty)$ $\cancel{C}$ D

$(-\infty, 7]$ D

$(-3, 7)$ D

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