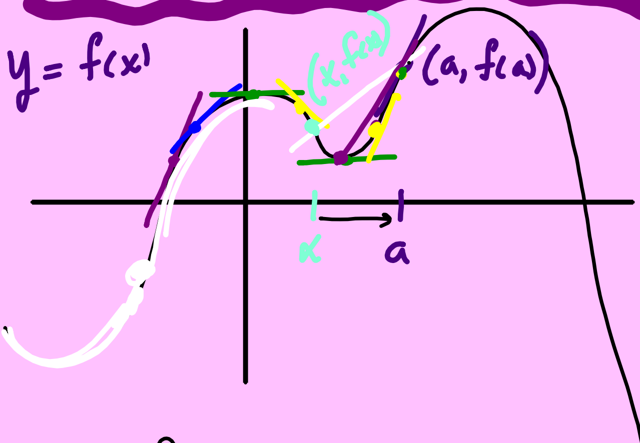
DERIVATIVES

- represents the slope of a line tangent to a curve at a given point.



$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

First Definition of Deriv.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Symbols

$f(x) =$	$f'(x) =$	$f''(x)$
$y =$	$y' =$	$y'' =$
	$\frac{dy}{dx} =$	$\frac{d^2y}{dx^2}$
	$D_x y$	

$$\frac{d^2}{dx^2} y$$

d

$$f(x) = x^3 - 2x^2 + x - 1 \qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to a} \frac{x^3 - 2x^2 + x \cancel{- 1} - (a^3 - 2a^2 + a \cancel{- 1})}{x - a} = \frac{0}{0}$$

$$\lim_{x \to a} \frac{(x^3 - a^3)(-2x^2 + 2a^2) + (x - a)}{x - a}$$

$$\lim_{x \to a} \frac{\cancel{(x-a)}(x^2 + ax + a^2) - 2\underbrace{\cancel{(x^2 - a^2)}}_{(x+a)\cancel{(x-a)}} + \cancel{(x - a)}}{\cancel{x - a}}$$

$$\lim_{x \to a} (x^2 + ax + a^2) - 2(x + a) + 1$$

$$= a^2 + a^2 + a^2 - 2(2a) + 1$$

$$f'(a) = \boxed{3a^2 - 4a + 1}$$

Find the equation of the tangent line at $x = 1$.

Point-Slope $(1, -1)$

$$f(x) = x^3 - 2x^2 + x - 1$$

$$y = 1^3 - 2(1)^2 + \cancel{x} - \cancel{x}$$

$$= 1 - 2$$

$$= -1$$

$$m = 3(1)^2 - 4(1) + 1$$

$$= 3 - 4 + 1$$

$$m = 0$$

$$y + 1 = 0(x - 1)$$

$$y + 1 = 0$$

$$\boxed{y = -1}$$

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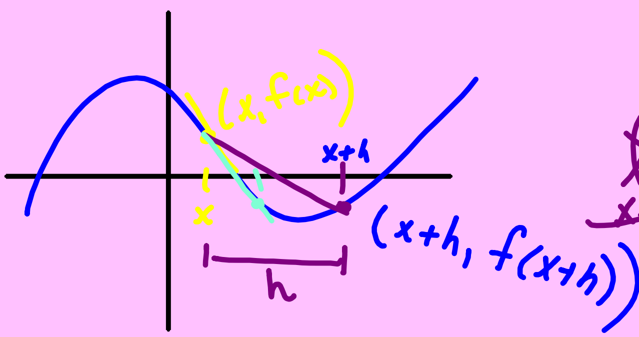
2ND DEFINITION OF THE DERIVATIVE



$f(x) = \sin x$

$$\cancel{\lim_{x \to a} \frac{\sin x - \sin a}{x - a}}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{\cancel{x} + h - \cancel{x}}$$

2nd Def.

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$f(x) = 3x^2 - 2x + 5$

$$\lim_{h \to 0} \frac{3(x+h)^2 - 2(x+h) + 5 - (3x^2 - 2x + 5)}{h}$$

$$3(x^2 + 2xh + h^2)$$

$$\lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h + \cancel{5} - \cancel{3x^2} + \cancel{2x} - \cancel{5}}{h}$$

$$\lim_{h \to 0} \frac{6xh + 3h^2 - 2h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}} = 6x + 0 - 2$$

$$= \boxed{6x - 2}$$

$f(x) = 8\sin x \qquad \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$\lim_{h\to 0} \frac{8\sin(\overset{A+B}{x+h}) - 8\sin x}{h}$$

$$8\lim_{h\to 0} \frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$

$$8\lim_{h\to 0} \frac{\sin x\cos h - \sin x}{h} + \frac{\cos x\sin h}{h}$$

$$8\lim_{h\to 0} -\sin x\frac{(1-\cos h)}{h} + \cos x \cdot \frac{\sin h}{h}$$

$$= 8\left[\cancel{-\sin x \cdot 0} + \cos x \cdot 1\right]$$

$$= 8\cos x$$

Deriv. of Trig Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$f(x) = 4\tan x + 3\csc x$$

$$f'(x) = 4\sec^2 x - 3\csc x \cot x$$

$$f(x) = 4\sec^8(x^3+4x)^9$$

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POWER RULE

$f(x)$	$f'(x)$
$x^3-2x^2+x-1x^0$	$3x^2-4x+1$
$3x^2-2x+5$	$6x-2$
$\frac{1}{x^2}=x^{-2}$	$-2x^{-3}=-\frac{2}{x^3}$

$$\frac{d}{dx}x^n=n\cdot x^{n-1}$$

$$f(x)=3x^8-\frac{1}{3x^5}-7\sqrt[3]{x^2}+31$$

$$=3x^8-\frac{1}{3}x^{-5}-7x^{2/3}+31$$

$$f'(x)=24x^7+\frac{5}{3}x^{-6}-\frac{14}{3}x^{-1/3}$$

$$=24x^7+\frac{5}{3x^6}-\frac{14}{3x^{1/3}}$$

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