LIMITS FROM GRAPHS

EXERCISE SET 2.3 Graphing Utility

FOCUS ON CONCEPTS

1–4 In these exercises, make reasonable assumptions about the end behavior of the indicated function.

1. For the function g graphed in the accompanying figure, find

(a) $\lim_{x\to-\infty} g(x)$ (b) $\lim_{x\to+\infty} g(x)$.

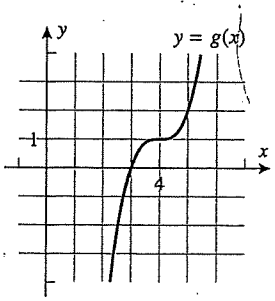

Figure Ex-1

2. For the function ϕ graphed in the accompanying figure, find

(a) $\lim_{x\to-\infty} \phi(x)$ (b) $\lim_{x\to+\infty} \phi(x)$.

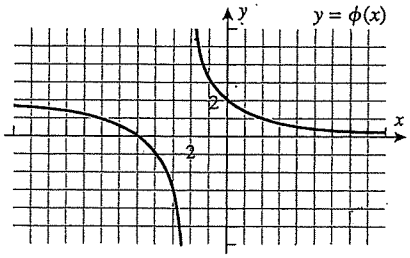

Figure Ex-2

3. For the function ϕ graphed in the accompanying figure, find

(a) $\lim_{x\to-\infty} \phi(x)$ (b) $\lim_{x\to+\infty} \phi(x)$.

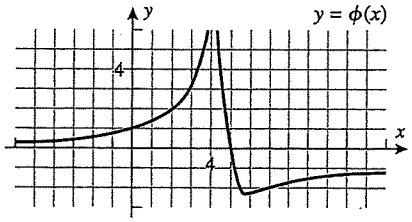

Figure Ex-3

4. For the function G graphed in the accompanying figure, find

(a) $\lim_{x\to-\infty} G(x)$ (b) $\lim_{x\to+\infty} G(x)$.

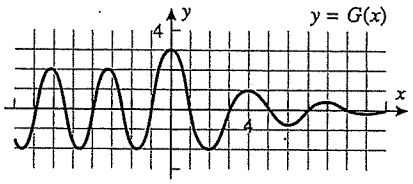

Figure Ex-4

5. Given that

$$\lim_{x\to+\infty} f(x) = 3, \quad \lim_{x\to+\infty} g(x) = -5, \quad \lim_{x\to+\infty} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to+\infty} [f(x) + 3g(x)]$

(b) $\lim_{x\to+\infty} [h(x) - 4g(x) + 1]$

(c) $\lim_{x\to+\infty} [f(x)g(x)]$ (d) $\lim_{x\to+\infty} [g(x)]^2$

(e) $\lim_{x\to+\infty} \sqrt[3]{5 + f(x)}$ (f) $\lim_{x\to+\infty} \dfrac{3}{g(x)}$

(g) $\lim_{x\to+\infty} \dfrac{3h(x) + 4}{x^2}$ (h) $\lim_{x\to+\infty} \dfrac{6f(x)}{5f(x) + 3g(x)}$

6. Given that

$$\lim_{x\to-\infty} f(x) = 7 \quad \text{and} \quad \lim_{x\to-\infty} g(x) = -6$$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to-\infty} [2f(x) - g(x)]$ (b) $\lim_{x\to-\infty} [6f(x) + 7g(x)]$

(c) $\lim_{x\to-\infty} [x^2 + g(x)]$ (d) $\lim_{x\to-\infty} [x^2 g(x)]$

(e) $\lim_{x\to-\infty} \sqrt[3]{f(x)g(x)}$ (f) $\lim_{x\to-\infty} \dfrac{g(x)}{f(x)}$

(g) $\lim_{x\to-\infty} \left[f(x) + \dfrac{g(x)}{x}\right]$ (h) $\lim_{x\to-\infty} \dfrac{xf(x)}{(2x + 3)g(x)}$

7–28 Find the limits.

7. $\lim_{x\to+\infty} (1 + 2x - 3x^5)$

8. $\lim_{x\to+\infty} (2x^3 - 100x + 5)$

9. $\lim_{x\to+\infty} \sqrt{x}$

10. $\lim_{x\to-\infty} \sqrt{5 - x}$

11. $\lim_{x\to+\infty} \dfrac{3x + 1}{2x - 5}$

12. $\lim_{x\to+\infty} \dfrac{5x^2 - 4x}{2x^2 + 3}$

13. $\lim_{y\to-\infty} \dfrac{3}{y + 4}$

14. $\lim_{x\to+\infty} \dfrac{1}{x - 12}$

15. $\lim_{x\to-\infty} \dfrac{x - 2}{x^2 + 2x + 1}$

16. $\lim_{x\to+\infty} \dfrac{5x^2 + 7}{3x^2 - x}$

17. $\lim_{x\to+\infty} \sqrt[3]{\dfrac{2 + 3x - 5x^2}{1 + 8x^2}}$

18. $\lim_{s\to+\infty} \sqrt[3]{\dfrac{3s^7 - 4s^5}{2s^7 + 1}}$

19. $\lim_{x\to-\infty} \dfrac{\sqrt{5x^2 - 2}}{x + 3}$

20. $\lim_{x\to+\infty} \dfrac{\sqrt{5x^2 - 2}}{x + 3}$

21. $\lim_{y\to-\infty} \dfrac{2 - y}{\sqrt{7 + 6y^2}}$

22. $\lim_{y\to+\infty} \dfrac{2 - y}{\sqrt{7 + 6y^2}}$

1. a) 0 b) 0 c) 0 d) 3

2. a) ∞ b) ∞ c) ∞ d) undef

3. a) $-\infty$ b) $-\infty$ c) $-\infty$ d) 1

4. a) 1 b) $-\infty$ c) DNE d) -2

5. -4,

6. 3

1. For the function F graphed in the accompanying figure, find
(a) $\lim_{x \to -2^-} F(x)$ (b) $\lim_{x \to -2^+} F(x)$
(c) $\lim_{x \to -2} F(x)$ (d) $F(-2)$.

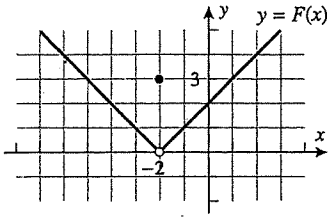

Figure Ex-1

2. For the function ϕ graphed in the accompanying figure, find
(a) $\lim_{x \to 4^-} \phi(x)$ (b) $\lim_{x \to 4^+} \phi(x)$
(c) $\lim_{x \to 4} \phi(x)$ (d) $\phi(4)$.

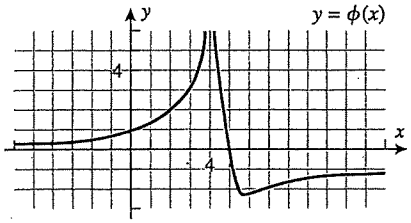

Figure Ex-2

3. For the function f graphed in the accompanying figure, find
(a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$
(c) $\lim_{x \to 3} f(x)$ (d) $f(3)$.

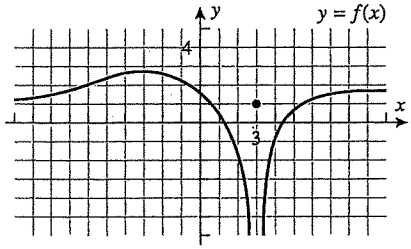

Figure Ex-3

4. For the function f graphed in the accompanying figure, find
(a) $\lim_{x \to 0^-} f(x)$ (b) $\lim_{x \to 0^+} f(x)$
(c) $\lim_{x \to 0} f(x)$ (d) $f(0)$.

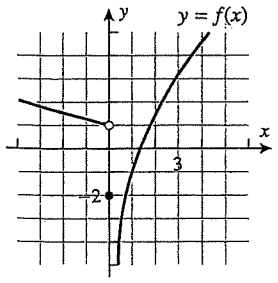

Figure Ex-4

5. Consider the function g graphed in the accompanying figure. For what values of x_0, $-7 \le x_0 \le 4$, does $\lim_{x \to x_0} g(x)$ not exist?

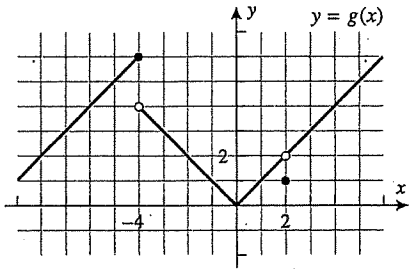

Figure Ex-5

6. Consider the function f graphed in the accompanying figure. For what values of x_0, $-9 \le x_0 \le 4$, does $\lim_{x \to x_0} f(x)$ not exist?

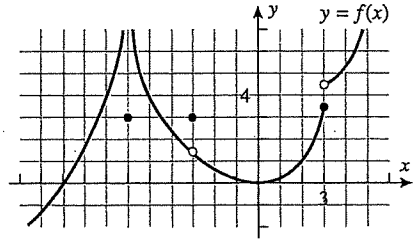

Figure Ex-6

FOCUS ON CONCEPTS

7–12 Sketch a possible graph for a function f with the specified properties. (Many different solutions are possible.)

7. (i) the domain of f is $[-1, 1]$
(ii) $f(-1) = f(0) = f(1) = 0$
(iii) $\lim_{x \to -1^+} f(x) = \lim_{x \to 0} f(x) = \lim_{x \to 1^-} f(x) = 1$

8. (i) the domain of f is $[-2, 1]$
(ii) $f(-2) = f(0) = f(1) = 0$
(iii) $\lim_{x \to -2^+} f(x) = 2$, $\lim_{x \to 0} f(x) = 0$, and $\lim_{x \to 1^-} f(x) = 1$

9. (i) the domain of f is $(-\infty, 0]$
(ii) $f(-2) = f(0) = 1$
(iii) $\lim_{x \to -2} f(x) = +\infty$

10. (i) the domain of f is $(0, +\infty)$
(ii) $f(1) = 0$
(iii) the y-axis is a vertical asymptote for the graph of f
(iv) $f(x) < 0$ if $0 < x < 1$

11. (i) $f(-3) = f(0) = f(2) = 0$
(ii) $\lim_{x \to -2^-} f(x) = +\infty$ and $\lim_{x \to -2^+} f(x) = -\infty$
(iii) $\lim_{x \to 1} f(x) = +\infty$

12. (i) $f(-1) = 0$, $f(0) = 1$, $f(1) = 0$
(ii) $\lim_{x \to -1^-} f(x) = 0$ and $\lim_{x \to -1^+} f(x) = +\infty$
(iii) $\lim_{x \to 1^-} f(x) = 1$ and $\lim_{x \to 1^+} f(x) = +\infty$

p. 131 1.(a) $-\infty$ (b) $+\infty$ 2. (a) 2 (b) 0 3. (a) 0 (b) −1 4. (a) DNE (b) 0

EXERCISE SET 2.3 Graphing Utility

FOCUS ON CONCEPTS

1–4 In these exercises, make reasonable assumptions about the end behavior of the indicated function.

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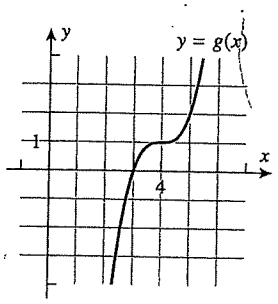

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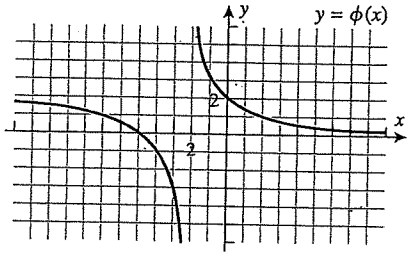

Figure Ex-2

3. For the function ϕ graphed in the accompanying figure, find
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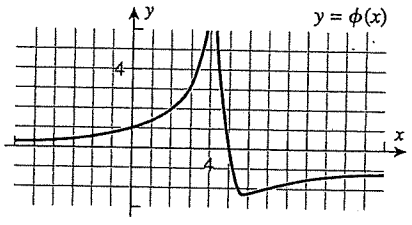

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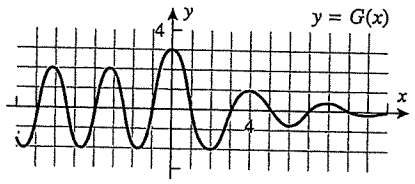

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5. Given that

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find the limits that exist. If the limit does not exist, explain why.

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(e) $\lim_{x\to-\infty} \sqrt[3]{f(x)g(x)}$ (f) $\lim_{x\to-\infty} \dfrac{g(x)}{f(x)}$

(g) $\lim_{x\to-\infty} \left[f(x) + \dfrac{g(x)}{x}\right]$ (h) $\lim_{x\to-\infty} \dfrac{xf(x)}{(2x+3)g(x)}$

7–28 Find the limits.

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9. $\lim_{x\to+\infty} \sqrt{x}$

10. $\lim_{x\to-\infty} \sqrt{5 - x}$

11. $\lim_{x\to+\infty} \dfrac{3x + 1}{2x - 5}$

12. $\lim_{x\to+\infty} \dfrac{5x^2 - 4x}{2x^2 + 3}$

13. $\lim_{y\to-\infty} \dfrac{3}{y + 4}$

14. $\lim_{x\to+\infty} \dfrac{1}{x - 12}$

15. $\lim_{x\to-\infty} \dfrac{x - 2}{x^2 + 2x + 1}$

16. $\lim_{x\to+\infty} \dfrac{5x^2 + 7}{3x^2 - x}$

17. $\lim_{x\to+\infty} \sqrt[3]{\dfrac{2 + 3x - 5x^2}{1 + 8x^2}}$

18. $\lim_{s\to+\infty} \sqrt[3]{\dfrac{3s^7 - 4s^5}{2s^7 + 1}}$

19. $\lim_{x\to-\infty} \dfrac{\sqrt{5x^2 - 2}}{x + 3}$

20. $\lim_{x\to+\infty} \dfrac{\sqrt{5x^2 - 2}}{x + 3}$

21. $\lim_{y\to-\infty} \dfrac{2 - y}{\sqrt{7 + 6y^2}}$

22. $\lim_{y\to+\infty} \dfrac{2 - y}{\sqrt{7 + 6y^2}}$

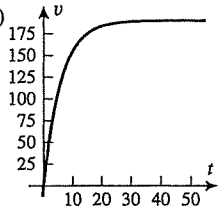
23. $\lim_{x\to-\infty} \dfrac{\sqrt{3x^4+x}}{x^2-8}$ **24.** $\lim_{x\to+\infty} \dfrac{\sqrt{3x^4+x}}{x^2-8}$

25. $\lim_{x\to+\infty} \dfrac{7-6x^5}{x+3}$ **26.** $\lim_{t\to-\infty} \dfrac{5-2t^3}{t^2+1}$

27. $\lim_{t\to+\infty} \dfrac{6-t^3}{7t^3+3}$ **28.** $\lim_{x\to-\infty} \dfrac{x+4x^3}{1-x^2+7x^3}$

ANSWERS

Exercise Set 2.3 (Page 131)

1. (a) $-\infty$ (b) $+\infty$ 3. (a) 0 (b) -1
5. (a) -12 (b) 21 (c) -15 (d) 25 (e) 2 (f) $-\frac{3}{5}$ (g) 0 (h) does not exist
7. $-\infty$ 9. $+\infty$ 11. $\frac{3}{2}$ 13. 0 15. 0 17. $\dfrac{-\sqrt[3]{5}}{2}$ 19. $-\sqrt{5}$
21. $1/\sqrt{6}$ 23. $\sqrt{3}$ 25. $-\infty$ 27. $-\frac{1}{7}$
29. $\lim_{t\to+\infty} n(t) = +\infty$; $\lim_{t\to+\infty} e(t) = c$ 31. (a) $+\infty$ (b) -5 33. 0
35. $a/2$ 37. $\lim_{x\to+\infty} p(x) = \begin{cases} -\infty, & n \text{ is odd} \\ +\infty, & n \text{ is even} \end{cases}$ and $\lim_{x\to-\infty} p(x) = +\infty$
39. For $m > n$, the limits are both zero; for $m = n$, the limits are equal to the leading coefficient of p; for $n > m$, the limits are $\pm\infty$
41. $\lim_{x\to-\infty} \dfrac{2+3x^n}{1-x^m} = \begin{cases} 0 & \text{if } m > n \\ -3 & \text{if } m = n \\ +\infty & \text{if } m < n \text{ and } n-m \text{ odd} \\ -\infty & \text{if } m < n \text{ and } n-m \text{ even} \end{cases}$
43. $+\infty$ 45. $+\infty$ 47. 1 49. 1 51. $-\infty$ 53. $-\infty$ 55. 1 57. $+\infty$
59. (a) 

(b) $c = 190$
(c) It is the terminal velocity of the skydriver.
61. They equal L. 63. e 65. e 67. $1/e$ 69. $+\infty$ 71. e 73. $1/e$
75. e^6 77. e^{-6} 79. $x+2$ 81. $1-x^2$ 83. $\sin x$

20. $-\sqrt{5}$

26. ∞

a) 2

b) ∞

c) $1/4$

d) $-\infty$

CONTINUITY

Using the 3-step process, determine whether the following functions are continuous or discontinuous at a.

1. $f(x) = \frac{2x^2-3}{x+4}$ $\quad a = -4$

2. $f(x) = \begin{cases} x^3 & \text{if } x < 2 \\ x^2 + 4 & \text{if } x \geq 2 \end{cases}$ $\quad a = 2$

3. $f(x) = \begin{cases} 2 - x & \text{if } x < -3 \\ 7 & \text{if } x = -3 \\ x^2 - 4 & \text{if } x \geq -3 \end{cases}$ $\quad a = -3$

4. $f(x) = \begin{cases} 3x + 5 & \text{if } x \leq 2 \\ 4x - 3 & \text{if } x > 2 \end{cases}$ $\quad a = 2$

Determine whether the given function is (C)ontinues or (D)iscontinuous on each interval.

5. $f(x) = \sqrt{4 - x^2}$ $\quad [-2,2]$ $\quad [2,3]$ $\quad (-2,2)$ $\quad (-1,5)$ $\quad (-\infty, 0]$

6. $g(x) = \frac{x+6}{x^2-36}$ $\quad (-\infty, 6]$ $\quad (-\infty, 4]$ $\quad (-6, \infty)$ $\quad (-6,6)$ $\quad (7, \infty)$

7. $h(x) = \frac{4x-3}{16x^2-9}$ $\quad \left[-\frac{3}{4}, 0\right]$ $\quad \left[-\frac{1}{2}, 0\right]$ $\quad \left(-\frac{3}{4}, \infty\right)$ $\quad [-2, \infty)$ $\quad \left(-1, -\frac{3}{4}\right)$

8. $k(x) = \frac{x}{\sqrt{3-x}}$ $\quad (-\infty, 3]$ $\quad (3, \infty)$ $\quad [-1, 3)$ $\quad [0,4]$ $\quad (1, 3]$

ANSWERS

1. $f(-4)$ is undefined.
   Discontinuous

2. $f(2) = 8$
   $\lim_{x\to 2^-} x^3 = 8$ $\lim_{x\to 2^+} x^2 + 4 = 8$ $\lim_{x\to 2} f(x) = 8$
   $f(2) = \lim_{x\to 2} f(x)$
   Continuous

3. $f(-3) = 7$
   $\lim_{x\to -3^-} 2 - x = 5$ $\lim_{x\to -3^+} x^2 - 4 = 5$ $\lim_{x\to -3} f(x) = 5$
   $f(-3) \neq \lim_{x\to -3} f(x)$
   Discontinuous

4. $f(2) = 11$
   $\lim_{x\to 2^-} 3x + 5 = 11$ $\lim_{x\to 2^+} 4x - 3 = 5$ $\lim_{x\to 2} f(x) = \text{DNE}$
   Discontinuous

5. C, D, C, D, D

6. D, D, D, C, C

7. D, C, D, D, C

8. D, D, C, D, D

ASYMPTOTES

Find the horizontal and vertical asymptotes of the graph of each function.

1. $f(x)=\dfrac{7x}{2x-5}$

2. $g(x)=\dfrac{3x^2+1}{2x^2-7x}$

3. $h(x)=\dfrac{-2x}{\sqrt{x^2+4}}$

4. $z(x)=\dfrac{x^2-1}{x}$

5. $k(x)=\dfrac{3x-7}{3x^2-4x-7}$

6. $p(x)=\dfrac{x^6+x^3-4}{1+2x^4}$

7. $f(x)=\dfrac{1-6x^2}{\sqrt[3]{x^9-1}}$

8. $h(x)=\dfrac{3x}{\sqrt{4x^2-1}}$

ANSWERS

1. $x=\dfrac{5}{2},\ y=\dfrac{7}{2}$

2. $x=0,\ x=\dfrac{7}{2},\ y=\dfrac{3}{2}$

3. $y=-2,\ y=2$

4. $x=0$

5. $x=-1,\ y=0$

6. None

7. $x=1,\ y=0$

8. $x=-\dfrac{1}{2},\ x=\dfrac{1}{2},\ y=-\dfrac{3}{2},\ y=\dfrac{3}{2}$

CAS WORKSHEET

Use CAS to complete each of the following operations.

Solve for x.

1. $\dfrac{2x}{2x+3}-\dfrac{2x}{2x-3}=1$

2. $\ln x+\ln(x+3)=5$

Evaluate the following limits.

3. $\lim\limits_{x\to 1}\dfrac{\ln x}{x-1}$

4. $\lim\limits_{x\to\infty}\dfrac{\ln\left(1+e^{x}\right)}{1+x}$

5. $\lim\limits_{x\to\frac{\pi}{2}}\left(\sec x-\tan x\right)$

6. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^{2}+13}}{8-2x}$

7. $\lim\limits_{x\to\pi^{-}}\csc x$

8. $\lim\limits_{x\to 2^{+}}\dfrac{x}{2-x}$

ANSWERS

1. $x=\dfrac{-3\left(\sqrt{2}+1\right)}{2}$ or $\dfrac{3\left(\sqrt{2}-1\right)}{2}$

2. $\dfrac{\sqrt{4e^{5}+9}-3}{2}\approx 10.77$

3. 1

4. 1

5. 0

6. $\dfrac{1}{2}$

7. $+\infty$

8. $-\infty$