More Chain Rule

$f(x) = \cos(3x^2 - 7x)$ $\quad f(x) = \cos x \ (3x^2 - 7x)$

$f'(x) = -\sin(3x^2 - 7x) \cdot (6x - 7)$

$\tan^2 x = (\tan x)^2$

$f(x) = \tan^8(x^5 - 3x^4)$

$= [\tan(x^5 - 3x^4)]^8$

$f'(x) = 8\tan^7(x^5 - 3x^4) \cdot \sec^2(x^5 - 3x^4) \cdot (5x^4 - 12x^3)$

$f(x) = \csc^5(\cot(3x^7)) = [\csc(\cot(3x^7))]^5$

$f'(x) = 5\csc^4(\cot(3x^7)) \cdot -\csc(\cot(3x^7))\cot(\cot(3x^7)) \cdot -\csc^2(3x^7) \cdot 21x^6$

$$f(x) = \csc^5(x^2) \cdot \cot(4x^8)$$

$$f'(x) = \csc^5(x^2) \cdot -\csc^2(4x^8) \cdot 32x^7 + \cot(4x^8) \cdot 5\csc^4(x^2) \cdot -\csc(x^2)\cot(x^2) \cdot 2x$$

$$f(x) = \tan\left(\sec\left(x^4 - 2x\right)^6\right)$$

$$f(x) = \sec^2\left(\sec\left(x^4-2x\right)^6\right) \cdot \sec\left(x^4-2x\right)^6 \tan\left(x^4-2x\right)^6 \cdot 6\left(x^4-2x\right)^5 \cdot \left(4x^3-2\right)$$

DIFFERENTIALS

$y = f(x)$

$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx} = f'(x)$

Find dy.

$y = x^3 - 3x^2 + 7$

$\frac{dy}{dx} = (3x^2 - 6x)\,dx$

$dy = (3x^2 - 6x)\,dx$

The radius of a sphere is measured to be 20 in. with a possible error of ± 0.3 in. Estimate the possible error in volume.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2 \, dr$$

$$dV = 4\pi (20 \text{ in})^2 (\pm 0.3 \text{ in})$$

$$dV = \pm 150.8 \text{ in}^3$$

% error

$$\frac{\Delta r}{r} = \frac{dr}{r} = \frac{\pm 0.3}{20}$$

$$= \pm 0.015$$

$$\approx 1.5\%$$

$$\frac{\Delta V}{V} = \frac{dV}{V} = \frac{4\pi r^2 \, dr}{\frac{4}{3}\pi r^3}$$

$$= 3\frac{dr}{r}$$

$$= 3(1.5\%)$$

$$= 4.5\%$$

September 28, 2023