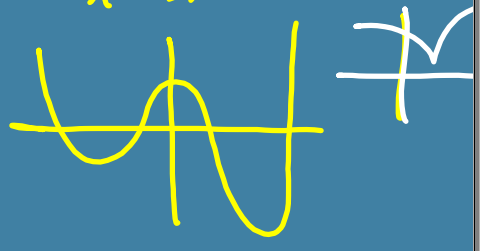
Polynomials & Rational Functions

Polynomials – many terms
– whole # exponents

$x^4 - 3x^3 + 2x^2 = 7$



Graph Characteristics

1) Smooth, rounded turns
2) continuous (no asymptotes, no holes, no sharp points, no end pts.)

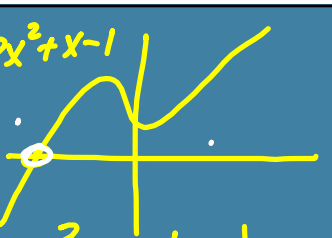
Degree – highest power

Max # of relative max/min
= Degree − 1

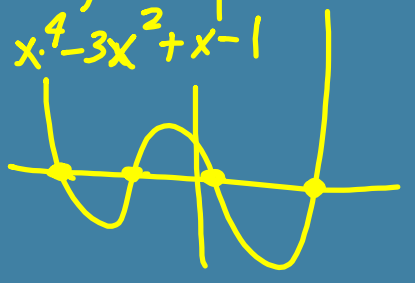
Max # of x-intercepts:
= Degree

zeros, roots, solutions

$f(x) = x^3 - 2x^2 + x - 1$
Degree = 3



$f(x) = x^4 - 3x^2 + x - 1$
$0 =$



End Behavior

even degree – both ends go in same direction
odd degree – ends go in opposite directions

$f(x) = -3x^6 + 7x^5 - 28x^3 + 2x^2 - 7$

Degree: 6

$\lim_{x \to -\infty} f(x) = -\infty$

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Max of Relative Max/Min: $6 - 1 = 5$

Max of Zeros: 6

$f(x) = -2x^5 + 7x^3 - 4x + 1$

leading coefficient ↪ determines direction of ends

Degree = 5

Max rel Max/Min = 4

Max Zeros = 5

$\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to +\infty} f(x) = -\infty$

Solving Polynomials

4 terms = grouping

$(2x^3 - 8x^2 + 3x - 12) = 0$

$2x^2(x-4) + 3(x-4) = 0$

$(x-4)(2x^2+3) =$

$x - 4 = 0$

$x = 4$

$\frac{2x^2}{2} = \frac{-3}{2}$

$\sqrt{x^2} = \sqrt{-3/2}$

$x = \frac{\sqrt{-3}}{\sqrt{2}} = \frac{i\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\pm i\sqrt{6}}{2}$

Solve.

$2x^5 + 3x^4 - x^3 + 9x^2 - 55x - 30 = 0$

$(2x\ \ 3)(x\ \ )(x\ \ )(x-2)$

$x = \frac{\pm 1, \pm 3, \pm 5, \pm 6, \pm 10, \pm 30}{\pm 1, \pm 2}$

↑ factors of leading coeff.

$2x^5 + 3x^4 - x^3 + 9x^2 - 55x - 30 \mid x = 1$

$x = 2 \quad 0$

2	2	3	-1	9	-55	-30
+		4	14	26	70	30
	2	7	13	35	15	0

$(x-2)(2x^4 + 7x^3 + 13x^2 + 35x + 15)$

$\pm 1, \pm 3, \pm 5, \pm 15$ ~~± 2~~

$2x^4 \ldots\ldots + 15 \mid x = -3 \quad 0$

-3	2	7	13	35	15
+		-6	-3	-30	-15
	2	1	10	5	0

$(x-2)(x+3)(2x^3 + x^2 + 10x + 5)$

$(x-2)(x+3)[x^2(2x+1) + 5(2x+1)]$

$(x-2)(x+3)(2x+1)(x^2+5)$

$x = 2, -3 \quad x = -\frac{1}{2}$

$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm i\sqrt{5}$

Steps

1) Find a number that make $f(x) = 0$ using factors of last term / factor of leading coeff.
2) Use # to perform synthetic division
3) Solve remaining factors
4) Repeat, if needed.

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