RADICAL EQUATIONS

Solve.

$\sqrt{3x+4} - \sqrt{x+2} = 2$

$\left(\sqrt{3x+4}\right)^2 = \left(2 + \sqrt{x+2}\right)^2$

$3x+4 = (2+\sqrt{x+2})(2+\sqrt{x+2})$

$3x+4 = 4 + 2\sqrt{x+2} + 2\sqrt{x+2} + x + 2$

$3x+4 = 6 + x + 4\sqrt{x+2}$

$-x \quad -6 \qquad -6 \quad -x$

$(2x-2)^2 = \left(4\sqrt{x+2}\right)^2$

$(2x-2)(2x-2)$

$4x^2 - 4x - 4x + 4 = 16(x+2)$

$4x^2 - 8x + 4 = 16x + 32$

$-16x - 32$

$4x^2 - 24x - 28 = 0$

$4(x^2 - 6x - 7) = 0$

$4(x-7)(x+1) = 0$

$x = 7$ ~~$x = -1$~~ Must Check.

$x=7$ Check: $\sqrt{25} - \sqrt{9} = 2$, $5 - 3 = 2$ ✓

$x=-1$ $\sqrt{1} - \sqrt{1} = 2$, $1 - 1 = 2$ No!

$y = \frac{2}{x-2} + 3$

Partial Fractions – purpose to find the original fractions that were combined to make the given fraction.

$$\frac{(x-4)}{(x-4)}\frac{2}{x+3} + \frac{6}{x-4}\frac{(x+3)}{(x+3)} = \frac{2x-8+6x+18}{(x+3)(x-4)} = \frac{8x+10}{(x+3)(x-4)}$$

$$\frac{8x+10}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$$

Multiply by common denom & cancel all denoms!

$$8x+10 = A(x-4) + B(x+3)$$

$$8x+10 = Ax - 4A + Bx + 3B$$

$$8 = A + B$$
$$10 = -4A + 3B$$

$$\begin{bmatrix} 1 & 1 \\ -4 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

$$\frac{2}{x+3} + \frac{6}{x-4}$$

$$\frac{\quad}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$\frac{\quad}{(x^2+4)(x^3+7)} = \frac{Ax^1+B}{x^2+4} + \frac{Cx^2+Dx+E}{x^3+7}$$

$$\frac{\quad}{(x-5)^2(x+3)} = \frac{A}{(x-5)^2} + \frac{B}{(x-5)^1} + \frac{C}{x+3}$$

$$\frac{\quad}{x^3(4x+1)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{4x+1}$$

$$\frac{A}{(x-0)^3}$$

$$\frac{10x^2+24x+8}{x^3+3x^2+4x+12}$$

$$x^3+3x^2+4x+12$$

$$x^2(x+3)+4(x+3)$$

$$(x+3)(x^2+4)$$

$$\frac{10x^2+24x+8}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4}$$

$$10x^2+24x+8 = A(x^2+4)+(x+3)(Bx+C)$$

$$10x^2+24x+8 = Ax^2+4A+Bx^2+Cx+3Bx+3C$$

$$10 = A+B$$
$$24 = 0+3B+C$$
$$8 = 4A+0+3C$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10 \\ 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

$$\frac{2}{x+3} + \frac{8x}{x^2+4}$$