DERIVATIVES OF EXPONENTIAL + LOG FUNCTIONS

$\ln x$, $\log_b x$, e^x, a^x, + x^x

$f(x) = e^x$

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$\lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$\lim_{h \to 0} e^x \frac{(e^h - 1)}{h}$$

$$= e^x \cdot 1$$

$$= e^x$$

$$\frac{d}{dx} e^x = e^x$$

$f(x) = e^{x^2+4x}$

$f'(x) = e^{x^2+4x} \cdot (2x+4)$

$f(x) = x^2 \cdot e^{4x^3}$

$$f'(x) = \underbrace{x^2 \cdot e^{4x^3} \cdot 12x^2} + \underbrace{e^{4x^3} \cdot 2x}$$

$$= 2xe^{4x^3}\left[6x^3 + 1\right]$$

$$e^{y} = e^{\ln x}$$

$$e^{y} = x \qquad \text{Find } \frac{dy}{dx}.$$

$$e^{y} \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^{y}}$$

$$= \frac{1}{x}$$

$$\frac{d}{dx} e^{x} = e^{x}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} a^{x} = \ln a \cdot a^{x}$$

$$\cancel{e^{x} = \ln e \cdot e^{x}}$$

$$y = a^{x}$$

$$\ln y = \ln a^{x}$$

$$\ln y = x \cdot \ln a$$

$$\frac{1}{y}\frac{dy}{dx} = \ln a$$

$$\frac{dy}{dx} = y \cdot \ln a$$

$$= a^{x} \cdot \ln a$$

$$f(x) = 7^{x}$$

$$f'(x) = \ln 7 \cdot 7^{x}$$

$$f(x) = 5^{x^2}$$

$$f'(x) = \ln 5 \cdot 5^{x^2} \cdot 2x$$

$$= 2\ln 5 \cdot x \cdot 5^{x^2}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} a^x = \ln a \cdot a^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$f(x) = 14^{\cos x}$$

$$f'(x) = \ln 14 \cdot 14^{\cos x} \cdot -\sin x$$

$$f(x) = 7^{\frac{e^{x^3}}{\tan x}}$$

$$f'(x) = \ln 7 \cdot 7^{\frac{e^{x^3}}{\tan x}} \cdot \left[\frac{\tan x \cdot e^{x^3} \cdot 3x^2 - e^{x^3} \cdot \sec^2 x}{\tan^2 x}\right]$$

$$= \ln 7 \cdot e^{x^3} \cdot 7^{\frac{e^{x^3}}{\tan x}} \left[\frac{3x^2 \tan x - \sec^2 x}{\tan^2 x}\right]$$

$$f(x) = \ln(x^3 - 4x^2)$$

$$f'(x) = \frac{1}{x^3 - 4x^2} \cdot (3x^2 - 8x)$$

$$= \frac{3x^2 - 8x}{x^3 - 4x^2}$$

$$= \frac{\cancel{x}(3x-8)}{\cancel{x}(x^2-4x)}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} a^x = \ln a \cdot a^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$f(x) = x^3 \cdot \sec(\ln \underline{x^2})$$

$$f'(x) = \underbrace{x^3 \cdot \underbrace{\sec(\ln x^2)\tan(\ln x^2) \cdot \frac{1}{x^2} \cdot 2x}} + \underbrace{\sec(\ln x^2) \cdot 3x^2}$$

$$= x^2 \sec(\ln x^2)\left[\cancel{x} \cdot \tan(\ln x^2) \cdot \frac{2}{\cancel{x}} + 3\right]$$

$$= x^2 \sec(\ln x^2)\left(2\tan(\ln x^2) + 3\right)$$

$f(x) = \log_8 3x^7$

$= \frac{\ln 3x^7}{\ln 8}$

$= \frac{1}{\ln 8} \cdot \ln 3x^7$

$f'(x) = \frac{1}{\ln 8} \cdot \frac{1}{\cancel{3}x^{\cancel{7}\,1}} \cdot \overset{7}{\cancel{21}}x^{\cancel{6}}$

$= \frac{7}{\ln 8 \cdot x} = \frac{7}{\ln 8} \cdot \frac{1}{x}$

Change of Base Formula

$\log_b a = \frac{\ln a}{\ln b}$

$\log_9 x^3 = \frac{\ln x^3}{\ln 9}$

$$f(x) = x^{x^2}$$

$$= e^{\ln x^{x^2}}$$

$$f(x) = e^{x^2 \cdot \ln x}$$

$$f'(x) = e^{x^2 \cdot \ln x} \cdot \left[x^{\cancel{2}\,1} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 2x \right]$$

$$= x^{x^2}\left[x + 2x \cdot \ln x \right]$$

$$= x^{x^2+1}\left[1 + 2\ln x \right]$$

Steps for derivative of $x^{f(x)}$ (tower func.)

1) Rewrite as $e^{\ln x^{f(x)}}$
2) Plog to $e^{f(x) \cdot \ln x}$
3) Perform the derivative (product rule)
4) Change $e^{f(x) \cdot \ln x}$ back to the original $x^{f(x)}$
5) Simplify + pull out common factors.

(example on next page)

$$f(x) = x^{\cos x} = e^{\ln x^{\cos x}} = e^{\cos x \cdot \ln x}$$

$$f'(x) = e^{\cos x \cdot \ln x} \cdot \left[\cos x \cdot \frac{1}{x} + \ln x \cdot -\sin x\right]$$

$$= x^{\cos x}\left[\frac{\cos x}{x} - \frac{x \cdot \sin x \ln x}{x \cdot 1}\right]$$

← make a common denom

$$= x^{\cos x}\left[\frac{\cos x - x\sin x \ln x}{x^1}\right]$$

$$= \frac{x^{\cos x}}{x^1}\left[\cos x - x\sin x \ln x\right]$$

$$= x^{\cos x - 1}\left[\cos x - x\sin x \ln x\right]$$