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Inverse Trig Functions

$$y = \sin^{-1} x$$

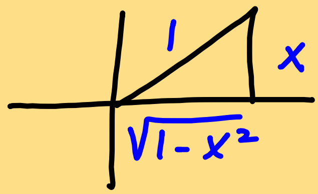
$$\Rightarrow \frac{x}{1} = \sin y$$

$$1 = \cos y \, \frac{dy}{dx}$$

$$\frac{1}{\cos y} = \frac{dy}{dx}$$

$$\frac{1}{\frac{\sqrt{1-x^2}}{1}} = \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$$



$$a^2 + x^2 = 1$$

$$\sqrt{a^2} = \sqrt{1-x^2}$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1} \qquad \frac{d}{dx}\cot^{-1}x = \frac{-1}{x^2+1}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$f(x) = \sin^{-1}(7x^5)$$

$$= \frac{1}{\sqrt{1-(7x^5)^2}} \cdot 35x^4 = \frac{35x^4}{\sqrt{1-49x^{10}}}$$

$$f(x) = \csc^{-1}(x^4)\ \tan^{-1}(\ln x^2)$$

$$f'(x) = \csc^{-1}(x^4) \cdot \frac{1}{(\ln x^2)^2+1} \cdot \frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x} + \tan^{-1}(\ln x^2) \cdot \frac{-1 \cdot 4\cancel{x^3}}{\cancel{x^4}_{x}\sqrt{x^8-1}}$$

$$= \frac{2\csc^{-1}(x^4)}{x\left[(\ln x^2)^2+1\right]} - \frac{4\tan^{-1}(\ln x^2)}{x\sqrt{x^8-1}}$$

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L'Hopital's Rule

Indeterminate Forms

$\frac{0}{0}$ $\frac{\infty}{\infty}$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{4-4}{2-2} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = 4$$

$$\lim_{x \to 2} \frac{2x}{1} = 4$$

L'Hopital's Rule

If $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

$$\lim_{x \to *} \frac{f(x)}{g(x)} = \lim_{x \to *} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 1} \frac{x^3-3x^2+5x-3}{x^2+x-2} = \frac{1-3+5-3}{1+1-2} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{3x^2-6x+5}{2x+1} = \frac{3-6+5}{2+1} = \frac{2}{3}$$

$$\lim_{x \to 1} \frac{x^3 - 3x^2 + 5x - 3}{x^2 + x - 2} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x - 1 - x}{\cos(2x) - 1} = \frac{1 - 1 - 0}{1 - 1} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x - 1}{-\sin(2x) \cdot 2} = \frac{1 - 1}{0 \cdot 2} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x - 1}{-2\sin(2x)}$$

$$\lim_{x \to 0} \frac{e^x}{-2\cos(2x) \cdot 2} = \frac{1}{-2 \cdot 1 \cdot 2}$$

$$= \boxed{\frac{1}{-4}}$$

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$$\lim_{x\to 0^+} \frac{1-\ln x}{e^{1/x}} \quad \Rightarrow x^{-1} \quad -1x^{-2}$$

$$\lim_{x\to -\infty} e^x = 0 \qquad \lim_{x\to\infty} e^x = \infty$$

$$\lim_{x\to 0^+} \ln x = -\infty \qquad \lim_{x\to\infty} \ln x = \infty$$

$$\lim_{x\to 0^+} \frac{1++\infty}{e^{1/0^+}} = \frac{\infty}{e^{+\infty}} = \frac{\infty}{\infty}$$

$$\lim_{x\to 0^+} \frac{-\frac{1}{x}}{e^{1/x}\cdot\frac{-1}{x^2}}$$

$$\lim_{x\to 0^+} +\frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{+e^{1/x}}$$

$$\lim_{x\to 0^+} \frac{x}{e^{1/x}} = \frac{0}{\infty} = \boxed{0}$$

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