Special Derivatives

– Implicit Differentiation = more than one variable: can not solve for y in terms of x.

Explicit

$y = 3x^2 + 7x - 4$

$\frac{dy}{dx} = 6x + 7$

Implicit

$y^2 + 3xy + 7 = 2 - 5y$

$y^2 + 3xy + 5y = -5$

$y^2 + x^3 + y^3 = 5$ Find $\frac{dy}{dx}$ ← "normal"

$(3x^2+7x-4)^2 + x^3 + (3x^2+7x-4)^3 = 5$

$2(3x^2+7x-4)' \cdot (6x+7) + 3x^2 + 3(3x^2+7x-4)^2 \cdot (6x+7) = 0$

$= 2y \cdot \frac{dy}{dx} + 3x^2 + 3y^2 \frac{dy}{dx} = 0$

$2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2$

$\frac{dy}{dx}(2y + 3y^2) = -3x^2$

$$\frac{dy}{dx} = \frac{-3x^2}{2y + 3y^2}$$

Find $\frac{dy}{dx}$.

$(3x^2)(y^3) + 4y^5 = 6\sin y + 8x^5$

$3x^2 \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 6x + 20y^4 \frac{dy}{dx} = 6\cos y \frac{dy}{dx} + 40x^4$

$9x^2y^2 \frac{dy}{dx} + 6xy^3 + 20y^4 \frac{dy}{dx} = 6\cos y \frac{dy}{dx} + 40x^4$

$\frac{dy}{dx}(9x^2y^2 + 20y^4 - 6\cos y) = 40x^4 - 6xy^3$

$\frac{dy}{dx} = \frac{40x^4 - 6xy^3}{9x^2y^2 + 20y^4 - 6\cos y}$

Find the eq. of the tangent line at $(1,0)$.

$m = \frac{40(1)^4 - 6(1)(0)^3}{9(1)^2(0) + 20(0)^4 - 6\cos 0} = \frac{40}{-6} = -\frac{20}{3}$

$y - 0 = -\frac{20}{3}(x-1)$

$\boxed{y = -\frac{20}{3}x + \frac{20}{3}}$

Find $\frac{dx}{dy}$. ← normal $\quad \frac{x^2}{y} = 4y^3 + 6x$

$$\frac{y^2 \cdot \left(y \cdot 2x\frac{dx}{dy} - x^2 \cdot 1\right)}{y^2} = \left(12y^2 + 6\frac{dx}{dy}\right)y^2$$

$$2xy\frac{dx}{dy} - x^2 = 12y^4 + 6y^2\frac{dx}{dy}$$

$$\frac{dx}{dy}(2xy - 6y^2) = 12y^4 + x^2$$

$$\frac{dx}{dy} = \frac{12y^4 + x^2}{2xy - 6y^2}$$

Find $\frac{da}{dp}$. (normal →) $3r^7 + 6a^5 - 4p = p^7$

$$21r^6\frac{dr}{dp} + 30a^4\frac{da}{dp} - 4 = 7p^6$$

$$30a^4\frac{da}{dp} = 7p^6 + 4 - 21r^6\frac{dr}{dp}$$

$$\frac{da}{dp} = \frac{7p^6 + 4 - 21r^6\frac{dr}{dp}}{30a^4}$$

Find $\frac{dy}{dt}$. $\frac{\Delta y}{\Delta t}$ $4x^3 + 2y^5 = \cos x$

$$12x^2\frac{dx}{dt} + 10y^4\frac{dy}{dt} = -\sin x\frac{dx}{dt}$$

$$\frac{10y^4\frac{dy}{dt}}{10y^4} = \frac{-12x^2\frac{dx}{dt} - \sin x\frac{dx}{dt}}{10y^4}$$

13/ $\sin(xy)$

$\cos(xy)\cdot[\quad]$

13/ $\sin(xy) = ???$

$\cos(xy) \cdot \left[x \cdot \frac{dy}{dx} + y \cdot 1 \right]$

$x\cos(xy)\frac{dy}{dx} + y\cos(xy)$

23/ $\sqrt{x^4 + y^2} = \ldots\ldots$

$(x^4 + y^2)^{1/2}$

$\frac{1}{2}(x^4 + y^2)^{-1/2} \cdot (4x^3 + 2y\frac{dy}{dx}) =$

$\frac{2x^3 + y\frac{dy}{dx}}{\sqrt{x^4 + y^2}} = \ldots\ldots$