More L'Hopital's Rule

$$\lim_{x \to 0^+} x^2 \cdot \ln x = 0 \cdot -\infty$$

$$\lim_{x \to 0^+} \frac{\ln x}{x^{-2}} = \frac{-\infty}{\infty}$$

$$\lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}}$$

$$\lim_{x \to 0^+} \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{3}\,2}}{-2}$$

$$\lim_{x \to 0^+} \frac{x^2}{-2} = \frac{0}{-2} = 0$$

<u>Indeterminate Forms</u>

$\frac{0}{0}$, ∞

$0 \cdot \infty$, $\infty - \infty$,

0^0, 1^∞, ∞^0

Must be rearranged using algebra or trig identities to be $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

$$\lim_{x\to 0^+}\left(\csc x - \frac{1}{x}\right) = \infty - \infty$$

$$\lim_{x\to 0^+}\left(\frac{x\cdot 1}{\sin x} - \frac{1\cdot \sin x}{x}\right)$$

$$\lim_{x\to 0^+}\frac{x - \sin x}{x\sin x} = \frac{0-0}{0\cdot 0} = \frac{0}{0}$$

$$\lim_{x\to 0^+}\frac{1-\cos x}{x\cdot\cos x + \sin x\cdot 1} = \frac{1-1}{0\cdot 1 + 0} = \frac{0}{0}$$

$$\lim_{x\to 0^+}\frac{\sin x}{x\cdot -\sin x + \cos x\cdot 1 + \cos x}$$

$$\lim_{x\to 0^+}\frac{\sin x}{-x\sin x + 2\cos x} = \frac{0}{0\cdot 0 + 2\cdot 1} = \frac{0}{2} = 0$$

$$\lim_{x\to\infty} x^{\frac{1}{x}} = \infty^{\frac{1}{\infty}} = \infty^0$$

$$\lim_{x\to\infty} e^{\ln x^{1/x}}$$

$$\lim_{x\to\infty} e^{\frac{1}{x}\cdot \ln x}$$

$$e^{\lim_{x\to\infty} \frac{1}{x}\cdot \ln x}$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0$$

$$= e^0 = \boxed{1}$$

$$\lim_{x\to 0^+} (\csc x)^{\sin x} = \infty^0$$

$$\lim_{x\to 0^+} e^{\ln (\csc x)^{\sin x}}$$

$$\lim_{x\to 0^+} e^{\sin x \cdot \ln(\csc x)}$$

$$\lim_{x\to 0^+} \sin x \cdot \ln(\csc x)$$

$$\lim_{x\to 0^+} \frac{\ln(\csc x)}{\csc x} = \frac{\ln(\infty)}{\infty} = \frac{\infty}{\infty}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\csc x}\cdot \cancel{-\csc x \cot x}}{\cancel{-\csc x \cot x}}$$

$$= \frac{1}{\infty} = 0$$

$$= e^0 = \boxed{1}$$

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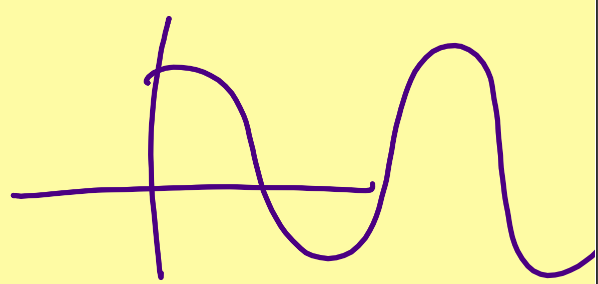
$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = (1+0)^{\infty} = 1^{\infty}$$

$$\lim_{x\to\infty} e^{\ln\left(1+\frac{1}{x}\right)^{x}}$$

$$\lim_{x\to\infty} e^{x\cdot\ln\left(1+\frac{1}{x}\right)}$$

$$\lim_{x\to\infty}\frac{\ln\left(1+\overset{x^{-1}}{\frac{1}{x}}\right)}{x^{-1}} = \frac{\ln\left(1+\overset{0}{\frac{1}{\infty}}\right)}{\frac{1}{\infty}} = \frac{\ln 1}{0} = \frac{0}{0}$$

$$\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{\frac{-1}{x^{2}}}}{\cancel{\frac{-1}{x^{2}}}}$$



$$= \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$$

$$\boxed{e^{1}}$$