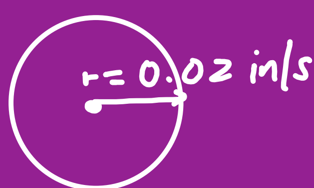
RELATED RATES

$\frac{mi}{h}$ $\frac{m}{s}$ $\frac{cm}{h}$ $\frac{gal}{min}$ $\frac{rad}{sec}$

- rate of one part of the situation impacts the rate of another part.

Example 1

$$\frac{d}{dt}\left[A = \pi r^2\right] \qquad \cancel{C = 2\pi r}$$



$$1 \cdot \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi \underset{in}{(4)} \underset{\frac{in}{sec}}{(0.02)}$$

$$\frac{dA}{dt} = 0.16\pi \ \frac{in^2}{sec}$$

$$\approx \boxed{0.5 \ \frac{in^2}{s}}$$

1) Draw a picture
2) Label with Variables (changing) & Constants (not changing)
3) Set up a formula
4) Do derivative with respect to time using implicit differentiation.
5) Identify the rate to be found.
6) Fill in values & solve.

2. $\frac{d}{dt}\left[V = \frac{4}{3}\pi r^3\right]$

$S.A = 0.64\pi$

$\frac{4\pi r^2}{4\pi} = \frac{0.64\cancel{\pi}}{4\cancel{\pi}}$

$\sqrt{r^2} = \sqrt{0.16}$

$r = 0.4$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-0.2\frac{m^3}{min} = 4\pi(0.4\,m)^2 \frac{dr}{dt}$

$\frac{-0.2}{0.64\pi} = \frac{0.64\pi}{0.64\pi}\frac{dr}{dt}$

$-0.0994 = \frac{dr}{dt}$

$\approx \boxed{-0.1\,\frac{m}{min}} = \frac{dr}{dt}$

3)

$h = 2r \Rightarrow \frac{h}{2} = r$

Find rate of height when 10 ft high

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{3}\pi \frac{h^2}{4} \cdot h$$

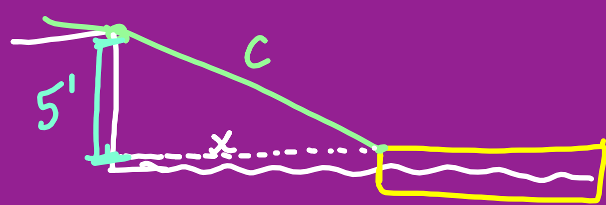
$$\frac{d}{dt}\left[V = \frac{1}{12}\pi h^3\right]$$

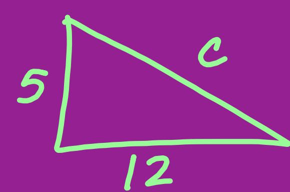
$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\underset{\frac{ft^3}{min}}{5} = \frac{1}{4}\pi \underset{ft^2}{(10)^2} \frac{dh}{dt}$$

$$\frac{5}{25\pi} = \frac{25\pi \frac{dh}{dt}}{25\pi}$$

$$\frac{1}{5\pi} \frac{ft}{min} = \frac{dh}{dt}$$





$$\frac{d}{dt}\left[c^2 = 25 + x^2\right]$$

$$2c\frac{dc}{dt} = 2x\frac{dx}{dt}$$

$$2\underset{ft}{(13)}\underset{ft/min}{(-4)} = 2\underset{ft}{(12)}\frac{dx}{dt}$$

$$\frac{-104}{24} = \frac{\cancel{24}\,\frac{dx}{dt}}{\cancel{24}}$$

$$-\frac{13}{3}\ ft/min = dx/dt$$

$$5^2 + 12^2 = c^2$$

$$169 = c^2$$

$$13 = c$$