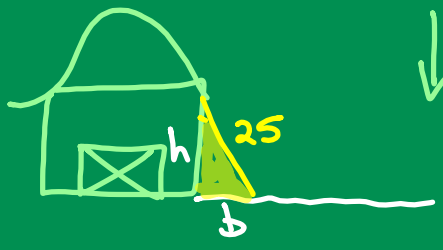
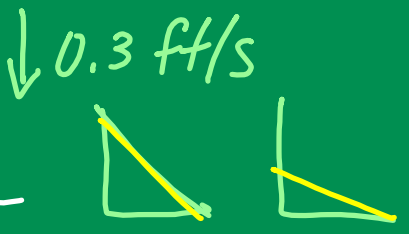
October 26, 2023

Related Rates 2

5/



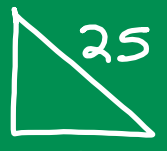


$A = \frac{1}{2}bh$

$\frac{dA}{dt} = \frac{1}{2}\left[b \cdot \frac{dh}{dt} + h \cdot \frac{db}{dt}\right]$

$\frac{dA}{dt} = \frac{1}{2}\left[7 \cdot -0.3 + 24 \cdot \left(\frac{36}{35}\right)\right]$

$\frac{dA}{dt} \approx 11.29 \ \frac{ft^2}{sec}$



$49 + h^2 = 625$

$\sqrt{h^2} = \sqrt{576}$

$h = 24$

$b^2 + h^2 = 25^2$

$2b\frac{db}{dt} + 2h\frac{dh}{dt} = 0$

$2b\frac{db}{dt} = -2h\frac{dh}{dt}$

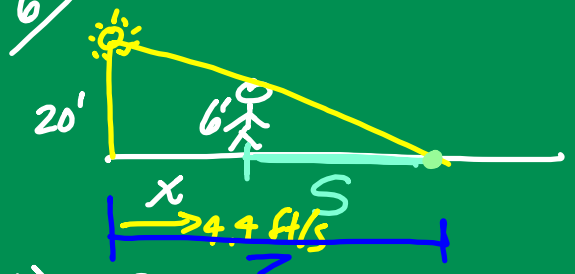
$2(7)\frac{db}{dt} = -2(24)(-0.3)$

$\frac{14\frac{db}{dt}}{14} = \frac{14.4}{14}$

$\frac{db}{dt} = \frac{144}{140} = \frac{72}{70}$

$= \frac{36}{35}$

6/



Similar Δ's

a) $\frac{20}{6} = \frac{x+s}{s}$

$20s = 6x + 6s$

$14s = 6x$

$14\frac{ds}{dt} = 6\frac{dx}{dt}$

$14\frac{ds}{dt} = 6(4.4)$

$14\frac{ds}{dt} = 26.4$

$ds/dt = 26.4/14$

$= \frac{66}{35} \approx \boxed{1.89 \text{ ft/s}}$

b) How fast is the tip of the shadow moving?

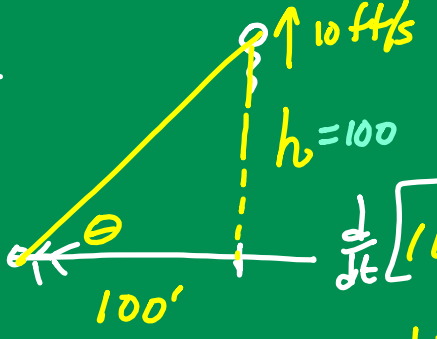
$z = x + s$

$\frac{dz}{dt} = \frac{dx}{dt} + \frac{ds}{dt}$

$\frac{dz}{dt} = 4.4 + 1.89$

$= 6.29 \text{ ft/s}$

1



$\tan\theta = \frac{h}{100}$

$\frac{d}{dt}\left[100\tan\theta = h\right]$

$100\sec^2\theta\frac{d\theta}{dt} = \frac{dh}{dt}$

$\sec\theta = \frac{r}{x} = \frac{\text{hyp}}{\text{adj}}$

$100\left(\sec 45\right)^2\frac{d\theta}{dt} = 10$

$100_{ft}\left(\sqrt{2}\right)^2\frac{d\theta}{dt} = 10$

$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

$200_{ft}\, d\theta/dt = 10\ ft/sec$

$\frac{d\theta}{dt} = \frac{10}{200} = \frac{1}{20}\frac{rad}{sec}$