CALCULUS HANDOUT
Related Rates

1. A rectangular swimming pool (with a horizontal bottom) is being drained. If its length and width are 25 ft and 20 feet and the water level is falling at the rate of ½ ft/min, how fast is the water draining?

2. The area of an isosceles right triangle decreases at the rate of 5 cm²/sec. At what rate are the legs changing when each has length 10 cm?

3. An icicle is in the shape of a right circular cone. At a certain point in time the height is 15 cm and is increasing at the rate of 1 cm/hr, while the radius of the base is 2 cm and is decreasing at 0.1 cm/hr. Is the volume of ice increasing or decreasing at that instant? At what rate?

4. A block of ice has a square top and bottom and rectangular sides. At a certain point in time each dimension of the square is 30 cm and is decreasing at 2 cm/hr, while the depth is 20 cm and is decreasing at 3 cm/hr. How fast is the ice melting?

5. A man 1.8 meters tall is 6 meters from the base of a streetlight that is 4 meters high. The man is walking directly away from the streetlight at the constant rate of 2 meters per second. (a) At what rate is his shadow lengthening? (b) At what rate is the tip of his shadow moving?

6. A 10-ft plank is leaning against a wall. If at a certain instant the bottom of the plank is 2 ft from the wall and is being pushed toward the wall at the rate of 6 in/s, how fast is the acute angle that the plank makes with the ground increasing?

7. Wheat is poured through a chute at the rate of 10 ft³/min, and falls in a conical pile whose bottom radius is always half the altitude. How fast will the circumference of the base be increasing when the pile is 8 feet high?

8. Water is flowing into a right circular conical tank 9 feet high and 6 feet across the top at the rate of 2 cubic feet per minute. At what rate is the water level rising when the water is 3 feet deep?

9. A horizontal eaves trough is 20 feet long and has a cross section in the shape of an isosceles triangle 8 inches across at the top and 10 inches deep. Because of a heavy rainstorm, the water in the trough is rising at the rate of ½ inch per minute at the instant when it is 5 inches deep. How fast is the volume of water in the trough increasing at this instant?

10. A water tank has the shape of an inverted right circular cone with a radius of 5 meters at the top and a height of 12 meters. At the instant when the water in the tank is 6 meters deep, more water is being poured in at the rate of 10 cubic meters per minute. Find the rate at which the surface level of the water is rising at this instant.

11. A water trough is 12 feet long, and its cross section is an equilateral triangle with sides 2 feet long. Water is pumped in the trough at a rate of 3 cubic feet per minute. How fast is the water level rising when the depth of the water is ½ foot?

ANSWERS

1. -250 ft³/min

2. $-\dfrac{1}{2}$ cm/sec

3. decreasing at $\dfrac{2\pi}{3}$ cm³/hr ≈ 2.09 cm³/hr

4. -5100 cm³/hr

5. (a) $\dfrac{18}{11}$ m/sec (b) $\dfrac{40}{11}$ m/sec

6. $\dfrac{1}{2\sqrt{96}} \approx 0.051$ rad/s

7. $\dfrac{5}{8}$ ft/min

8. $\dfrac{2}{\pi}$ ft/min

9. 480 in³/min

10. $\dfrac{8}{5\pi}$ m/min

11. $\dfrac{\sqrt{3}}{4}$ ft/min

RELATED RATES
EXAMPLE PROBLEMS

1. A circular metal plate expands when heated. The radius increases at a rate of 0.02 in/sec. At what rate is the surface area increasing when the radius is 4 inches long?

2. A spherical snowball is melting so that its volume is decreasing at the rate of 0.2 cubic meters per minute. Find the rate at which the radius is decreasing at the instant when the surface area is 0.64π m^2.

3. Sand is falling at the rate of 5 ft^3/min upon the tip of a sand pile that maintains the form of a right circular cone whose height is twice the radius of the base. Find the rate at which the height of the pile is increasing at the instant when the pile is 10 feet high.

4. A barge whose deck is 5 feet below the level of a dock is drawn up to it by means of a cable running over a pulley at the edge of the dock. When the barge is 12 feet away from the dock, the cable is being hauled in at the rate of 4 ft/min. At what rate is the barge moving at this time?

5. A 25 foot ladder is sliding down the side of a barn at the rate of 0.3 ft/sec. How fast is the area formed by the ground, ladder, and house changing when the base of the ladder is 7 feet from the house?

6. A man 6 feet tall is 12 feet from the base of lamp post that is 20 feet high. The man is walking directly away from the lamp post at the constant rate of 4.4 ft/sec. (a) At what rate is his shadow lengthening? (b) How fast is the tip of his shadow moving?

7. A balloon rises at the rate of 10 ft/sec from a point on the ground 100 feet from an observer. How fast is the angle of elevation changing when the balloon is 100 feet above the ground?

8. A train is traveling at 4/5 km/min along a straight track. A movie camera positioned 1 km from the track is focused on the train. When the train is 2 km from the camera, the angle formed at the camera is $\pi/3$ radians. How fast is the camera rotating at the moment when the train is 2 km from the camera?

9. Water is running out of a conical funnel at the rate of 3 in^3/sec. The funnel has a radius of 2 inches and a height of 8 inches. How fast is the water level dropping when it is 3 inches from the top?

10. A triangular trough is 12 feet long, 3 feet wide at the top, and 3 feet deep. If water is poured into the trough at a rate of 10 ft^3/min, how fast is the surface rising when the depth is 2 feet?