Special Derivatives Review

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} a^x = \ln a \cdot a^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\log_b a = \frac{\ln a}{\ln b}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$(2e^x y^2) + 4\sec(3x) = 7^{2y} - 46$ Find $\frac{dy}{dx}$.

$2e^x \cdot 2y\frac{dy}{dx} + y^2 \cdot 2e^x + 4\sec(3x)\tan(3x)\cdot 3 = \ln 7 \cdot 7^{2y} \cdot 2\frac{dy}{dx}$

$4e^x y\frac{dy}{dx} + 2y^2e^x + 12\sec(3x)\tan(3x) = 2\ln(7)\cdot 7^{2y}\frac{dy}{dx}$

$\frac{dy}{dx}\left(4e^x y - 2\ln 7 \cdot 7^{2y}\right) = -2y^2e^x - 12\sec(3x)\tan(3x)$

$\frac{dy}{dx} = \frac{-2y^2e^x - 12\sec(3x)\tan(3x)}{4e^x y - 2\ln 7 \cdot 7^{2y}}$

$m = \frac{y^2e^x + 6\sec(3x)\tan(3x)}{\ln(7)\cdot 7^{2y} - 2e^x y}$ $(0, 5)$

$m = \frac{25 + 6\cdot 0}{\ln(7)\cdot 7^{10} - 10} = \frac{25}{\ln 7 \cdot 7^{10} - 10} \approx 0$

$y - y_1 = m(x - x_1)$

$y - 5 = 0(x - 0)$

$y = 5$

$$f(x) = x^{\sec x} = e^{\ln x^{\sec x}} = e^{\sec x \cdot \ln x}$$

~~$\pm \sqrt{\sec x} \cdot x^{\sec x - 1}$~~

$$f(x) = e^{\sec x \cdot \ln x}$$

$$f'(x) = e^{\sec x \cdot \ln x} \cdot \left[\sec x \cdot \frac{1}{x} + \ln x \cdot \sec x \tan x \right]$$

$$= \sec x \cdot x^{\sec x} \left[\frac{1}{x} + \frac{\ln x \tan x}{x} \right]$$

$$= \sec x \cdot x^{\sec x} \left[\frac{1 + x \ln x \tan x}{x} \right]$$

$$f(x) = \csc^{-1}\left(\sqrt{x^2 - 1}\right)$$

$$f'(x) = \frac{-1}{\sqrt{x^2-1}\sqrt{x^2-1-1}} \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2-1} \cdot \sqrt{x^2-1} \cdot \sqrt{x^2-2}}$$

$$= \frac{x}{(x^2-1)\sqrt{x^2-2}}$$

$$f(x) = \frac{\log_6(4x^2)}{2x^4} = \frac{\frac{\ln(4x^2)}{\ln 6}}{2x^4} \cdot \frac{1}{2x^4} = \frac{1}{\ln 6} \cdot \frac{\ln(4x^2)}{2x^4}$$

$$f'(x) = \frac{1}{\ln 6} \cdot \left[\frac{2x^{\cancel{4}^{3}} \cdot \frac{1}{\cancel{4x^2}} \cdot \cancel{8}^{2}x - \ln(4x^2) \cdot 8x^3}{(2x^4)^2} \right]$$

$$\frac{1}{\ln 6} \left[\frac{4x^3 - \ln(4x^2) \cdot 8x^3}{4x^8} \right]$$

$$\frac{1}{\ln 6} \cdot \frac{\cancel{4}x^3(1 - 2\ln(4x^2))}{\cancel{4}x^{8_5}}$$