Confidence Intervals

$z = \frac{x - \mu}{\sigma}$

Inferential Statistics – use a sample to represent a population

Confidence Interval – an interval about the sample mean ($\bar{x}$) in which the population mean (μ) lies within a certain level of confidence.

90%
95%
99%

$\bar{x} = 25$

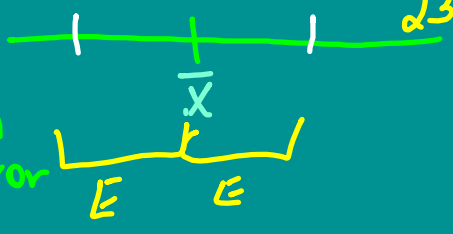
$\bar{x} \pm E$

25 ± 2

$23 - 27$

$\bar{x} \pm E$

↑ margin of error



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3 Steps to find a confidence interval.

1) Find standard deviation of the sampling distribution. (Standard error of the mean = $\sigma_{\bar{x}}$)

Pop. → $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}}$ ← if $n \geq 30$

Sample mean

2) Find margin of error (E)

$E = z \cdot \sigma_{\bar{x}}$

z is based on % confident

3) Find conf. interval

$\bar{x} \pm E$

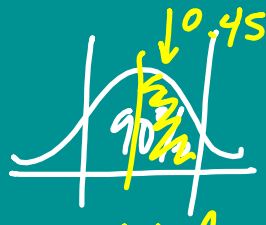
Mean weight of 36 h.s. wrestlers is 136.4 lb.
Standard dev = 14.1 lb. Find a 90% conf. interval.
for the mean weight of all h.s. wrestlers.

1) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14.1}{\sqrt{36}} = 2.35$ (use 3 decimal places)

2) Margin of Error $E = z \cdot \sigma_{\bar{x}}$

$E = 1.65 \cdot 2.35$

$E = 3.88$ lb.



Col. A

z	%
1.64	0.4495
1.65	0.4505

3) Confidence Interval

$\bar{x} \pm E$

136.4 lb $\pm$ 3.88 lb.

Conf. Intvl: 132.52 lb – 140.28 lb.

We are 90% confident that the mean of all high school wrestlers is between 132.52 lb & 140.28 lb.

81 cattle fed a special diet

Mean wt. gain = 105 lb. $s = 10$ lb.

What is the probability a cow gained 102 lb - 108 lb.? ← % conf

$E = 3$ lb.

$E = Z \cdot \sigma_{\bar{x}}$

$3 = Z \cdot 1.111$

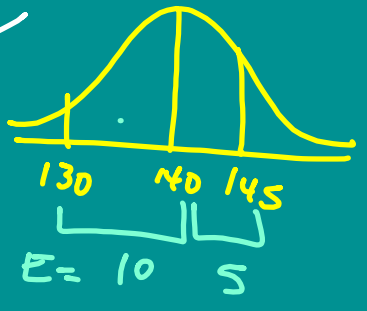
$\frac{3}{1.111} = Z$

$2.70 = Z$

$0.4965 * 2 = \boxed{0.993}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{81}} = 1.111$

Find prob. 130 — 145 $\bar{x} = 140$



Sample Size

$$n = \left(\frac{z \cdot \sigma}{E} \right)^2$$

- n: sample #
- z: based on Confidence
- E: margin of Error
- σ: St. dev. of pop.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$E = z \cdot \sigma_{\bar{x}} \Rightarrow \frac{E}{z} = \sigma_{\bar{x}}$$

$$\sqrt{n} \cdot \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{n}$$

$$\sqrt{n} \cdot \sigma_{\bar{x}} = \sigma$$

$$\left(\sqrt{n}\right)^2 = \left(\frac{\sigma}{\sigma_{\bar{x}}}\right)^2$$

$$n = \left(\frac{\sigma}{\frac{E}{z}} \right)^2$$

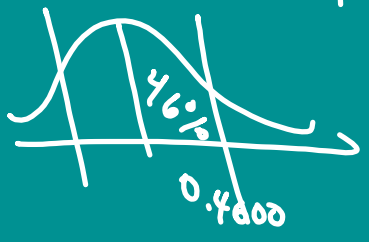
$$n = \left(\frac{\sigma \cdot z}{E} \right)^2$$

What sample size if:

92% prob.

Mean GPA 3.1 – 3.5 ($\frac{1}{3.3}$ 0.2)

$\sigma = 1.4$



$$n = \left(\frac{1.4 \cdot 1.75}{0.2} \right)^2 = 150.062$$

$\approx$ 151 student