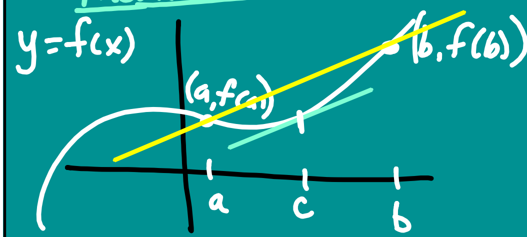
# CURVE SKETCHING

## Mean Value Theorem for Derivatives




1) $f$ is continuous $[a,b]$
2) $f$ is differentiable on $(a,b)$
3) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$f(x) = x^3 - 3x^2 + 2x \quad [\overset{a}{0}, \overset{b}{2}]$

Use Mean Value Thm to find point c.

1) $f$ is continuous (polyn.)
2) $f$ is differentiable (polyn)
3) $f'(x) = 3x^2 - 6x + 2$ $\qquad \dfrac{f(b)-f(a)}{b-a}$

$$3c^2 - 6c + 2 = \frac{f(2)-f(0)}{2-0}$$

$f(2) = 8 - 12 + 4 = 0$
$f(0) = 0 - 0 + 0 = 0$

$$3c^2 - 6c + 2 = \frac{0-0}{2-0}$$

$$3c^2 - 6c + 2 = 0$$

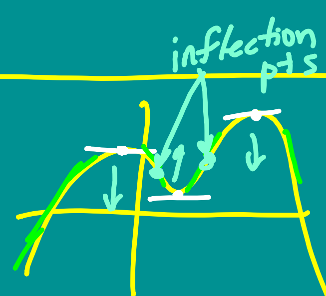
$$c = \frac{6 \pm \sqrt{36 - \underbrace{4(3)(2)}_{24}}}{2(3)}$$

$$c = \frac{6 \pm \sqrt{12}}{6}$$

$$c = \frac{6 \pm 2\sqrt{3}}{6} =$$

$$c = \frac{3 \cancel{+} \sqrt{3}}{3} \approx \begin{matrix} 1.6 \\ 4 \end{matrix}$$

$$c = \frac{3-\sqrt{3}}{3}$$




$f'(x) = 0$ critical points

inflection pts — where graph changes concavity
— use $f''$.

$f(x) = x^4 - 4x^3 + 10$

1) Find crit pts.

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0, 3$

$f'$: $+\cdot-$ $\quad +\cdot-$ $\quad +\cdot+$

$-$ | $-$ | $+$

$-1$ $\quad 0$ $\quad 1$ $\quad 3$ $\quad 60$

Decreasing $(-\infty, 0)(0, 3)$

Increasing $(3, \infty)$

$f''(x) = 12x^2 - 24x$

$0 = 12x^2 - 24x$

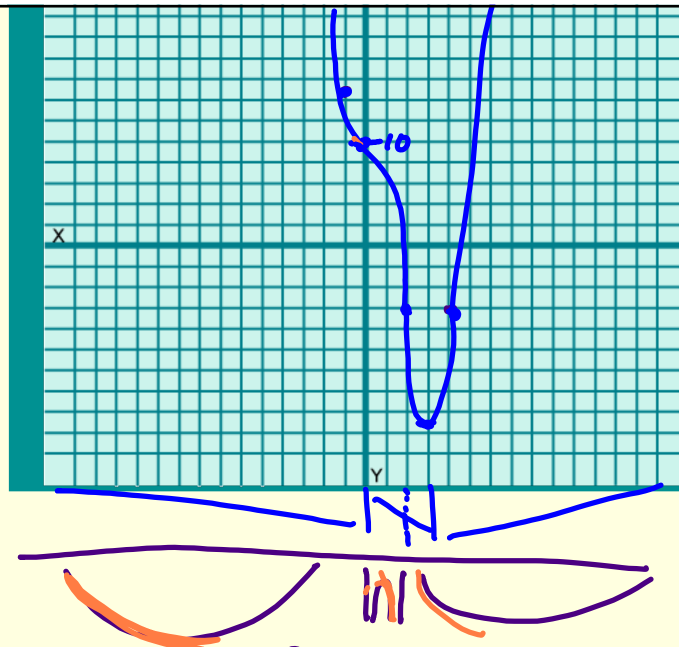
$\Rightarrow 0 = 12x(x-2)$

$x = 0, 2$ ← possible infl. pts.

$+\cdot-$

$+$ | $-$ | $+$

$0$ $\quad 1$ $\quad 2$




| $x$ | $f(x)$ |
|---|---|
| 0 | 10 |
| 2 | -6 |
| 3 | -17 |
| -1 | 15 |
| 4 | 10 |

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |

November 8, 2023

$f(x) = 3x^{2/3} - x$

1) Find critical pts.

$f'(x) = 2x^{-1/3} - 1$

$0 = \boxed{\frac{2}{\sqrt[3]{x}} - 1} \Leftarrow \sqrt[3]{x} = 0$

$x = 0$

pt. of non-differentiability

Where $f'$ is undef, but $f$ is defined.

$\sqrt[3]{x} \quad 1 = \frac{2}{\sqrt[3]{x}} \cdot \sqrt[3]{x}$

$(\sqrt[3]{x})^3 = (2)^3$

$x = 8$

$\frac{2}{-1} - 1 \quad \frac{2}{1} - 1 \quad \frac{2}{3} - 1$

$-$ | $+$ | $-$

$-1 \quad 0 \quad 1 \quad 8 \quad 27$

2) Check concavity

$f'(x) = 2x^{-1/3} - 1$

$f''(x) = -\frac{2}{3}x^{-4/3} = 0$

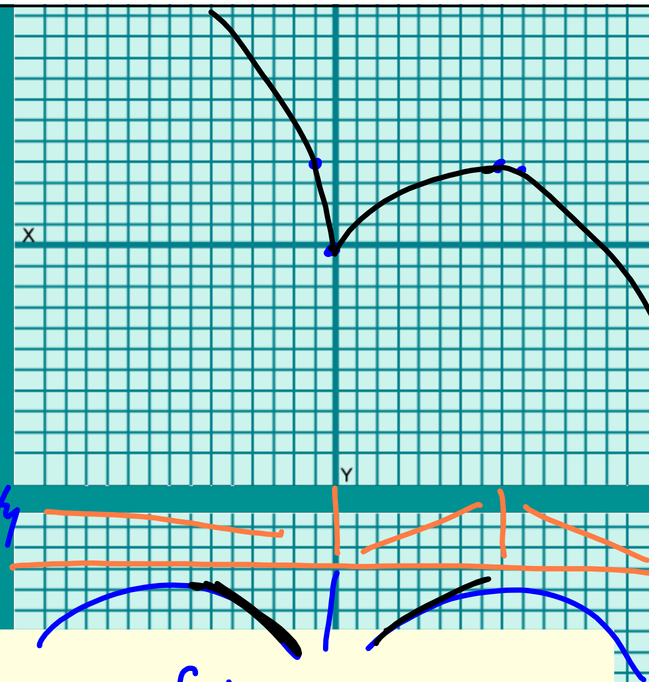
$= \frac{-2}{3\sqrt[3]{x^4}} = 0$

$\Longrightarrow -2 = 0$

Undef at $x = 0$

$-$ | $-$

$0$



| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 8 | 4 |
| -1 | 4 |
| 9 | 3.98 |

$\underline{\quad\quad} \mid x = \underline{\quad}$