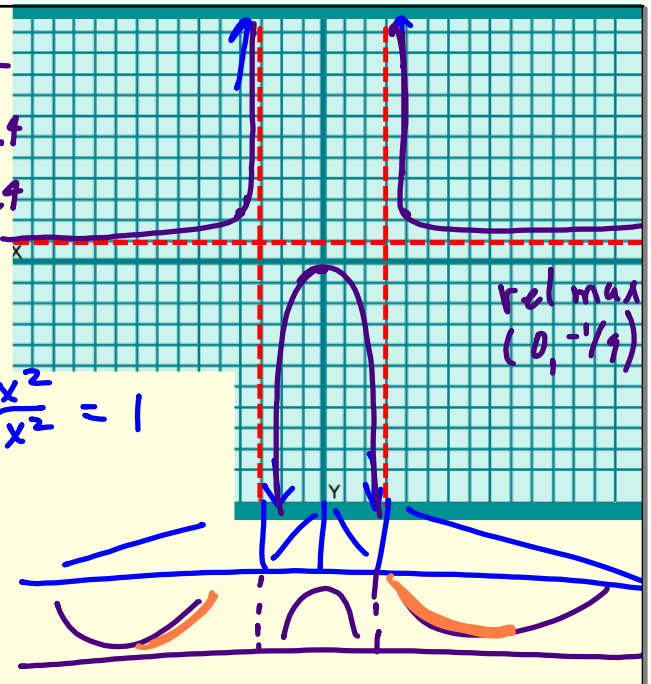
# Curve Sketching 2

$f(x) = \dfrac{x^2+1}{x^2-9}$ ⟸

| | |
|---|---|
| 0 | $-1/9$ |
| $-4$ | $\frac{17}{7} \approx 2.4$ |
| 4 | $\frac{17}{7} \approx 2.4$ |




**Vert.**

$\lim_{x\to 3} \dfrac{x^2+1}{x^2-9} = \dfrac{10}{0} = \dfrac{+}{-} = -\infty$

$\lim_{x\to 3^+} \dfrac{x^2+1}{x^2-9} = \dfrac{+}{+} = +\infty$

$\lim_{x\to -3^-} \dfrac{x^2+1}{x^2-9} = \dfrac{+}{+} = +\infty$

$\lim_{x\to -3^+} \dfrac{x^2+1}{x^2-9} = \dfrac{+}{-} = -\infty$

**Horiz**

$\lim_{x\to\pm\infty} \dfrac{x^2}{x^2} = 1$

$f(x) = \dfrac{x^2+1}{x^2-9}$

$f'(x) = \dfrac{(x^2-9)\cdot 2x - (x^2+1)\cdot 2x}{(x^2-9)^2}$

$= \dfrac{2x[x^2-9-x^2-1]}{(x^2-9)^2}$

$0 = \dfrac{-20x}{(x^2-9)^2}$ ⟵

$0 = -20x$

$0 = x$

| | | | |
|---|---|---|---|
| $\frac{+}{+}$ | $\frac{+}{+}$ | $\frac{=}{+}$ | $\frac{=}{+}$ |
| + | + | − | − |

$-3 \quad 0 \quad 3$

$f''(x) = \dfrac{(x^2-9)^2\cdot -20 - -20x\cdot 2(x^2-9)\cdot 2x}{(x^2-9)^4}$

$= \dfrac{-20\cdot(x^2-9)[x^2-9-4x^2]}{(x^2-9)^{4-3}}$

$= \dfrac{-20(-3x^2-9)}{(x^2-9)^3}$

$0 = \dfrac{60(x^2+3)}{(x^2-9)^3}$ ⟸

$x^2+3=0$

$\sqrt{x^2} = \pm\sqrt{-3}$

| | | | |
|---|---|---|---|
| $\frac{+}{+}$ | $\frac{+}{=}$ | $\frac{+}{+}$ | |
| + | − | + | |

$-4 \quad -3 \quad 0 \quad 3 \quad 4$

November 10, 2023

## Vertical Asymp

Denom = 0

$\lim_{x \to \#} f(x) = \pm\infty$

## Horiz Asymp

$\lim_{x \to \pm\infty} f(x) = \#$

$f(x) = x \cdot e^{-2x} = \frac{x}{e^{2x}}$ ✓

| | |
|---|---|
| 1/2 | 0.18 |
| 1 | 0.13 |
| 0 | 0 |
| 2 | .037 |

**Vertical**

None
$e^x \neq 0$

**Horiz**

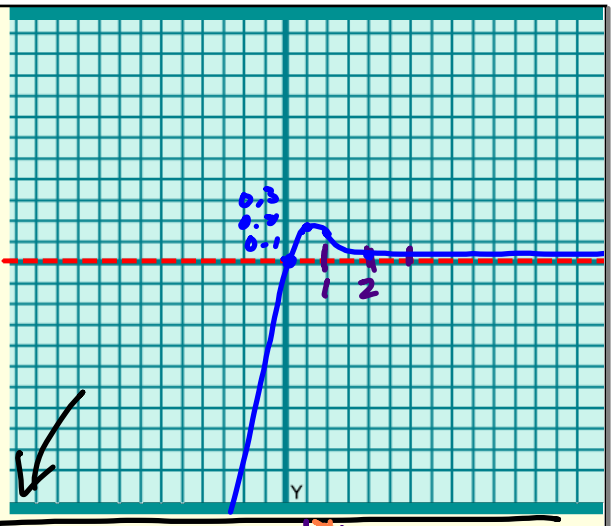
$\lim_{x \to +\infty} x \cdot e^{-2x} = \infty \cdot 0$

$\lim_{x \to +\infty} \frac{x}{e^{2x}} = \lim_{x \to \infty} \frac{1}{e^{2x} \cdot 2}$

$= \frac{1}{\infty} = 0$

$\lim_{x \to -\infty} x \cdot e^{-2x} = -\infty \cdot \infty$

$= -\infty$




$f'(x) = x \cdot e^{-2x} \cdot -2 + e^{-2x} \cdot 1$

$\Rightarrow = e^{-2x}(-2x + 1)$

$e^{-2} = \frac{1}{e^2}$

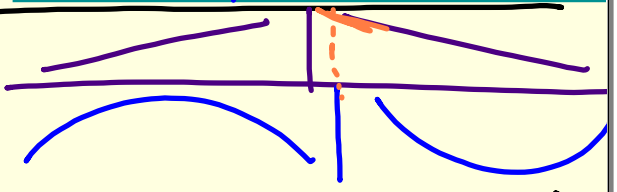
$-2x + 1 = 0$

$1 = 2x$

$1/2 = x$

+ · + | + · −

0 | 1/2 | 1



$f''(x) = e^{-2x} \cdot -2 + (-2x+1)e^{-2x} \cdot -2$

$= -2e^{-2x}(1 - 2x + 1)$

$= -2e^{-2x}(2 - 2x)$

$\Rightarrow = 4e^{-2x}(x - 1)$

$x = 1$

+ · − | + · +

0 | 1 | 2