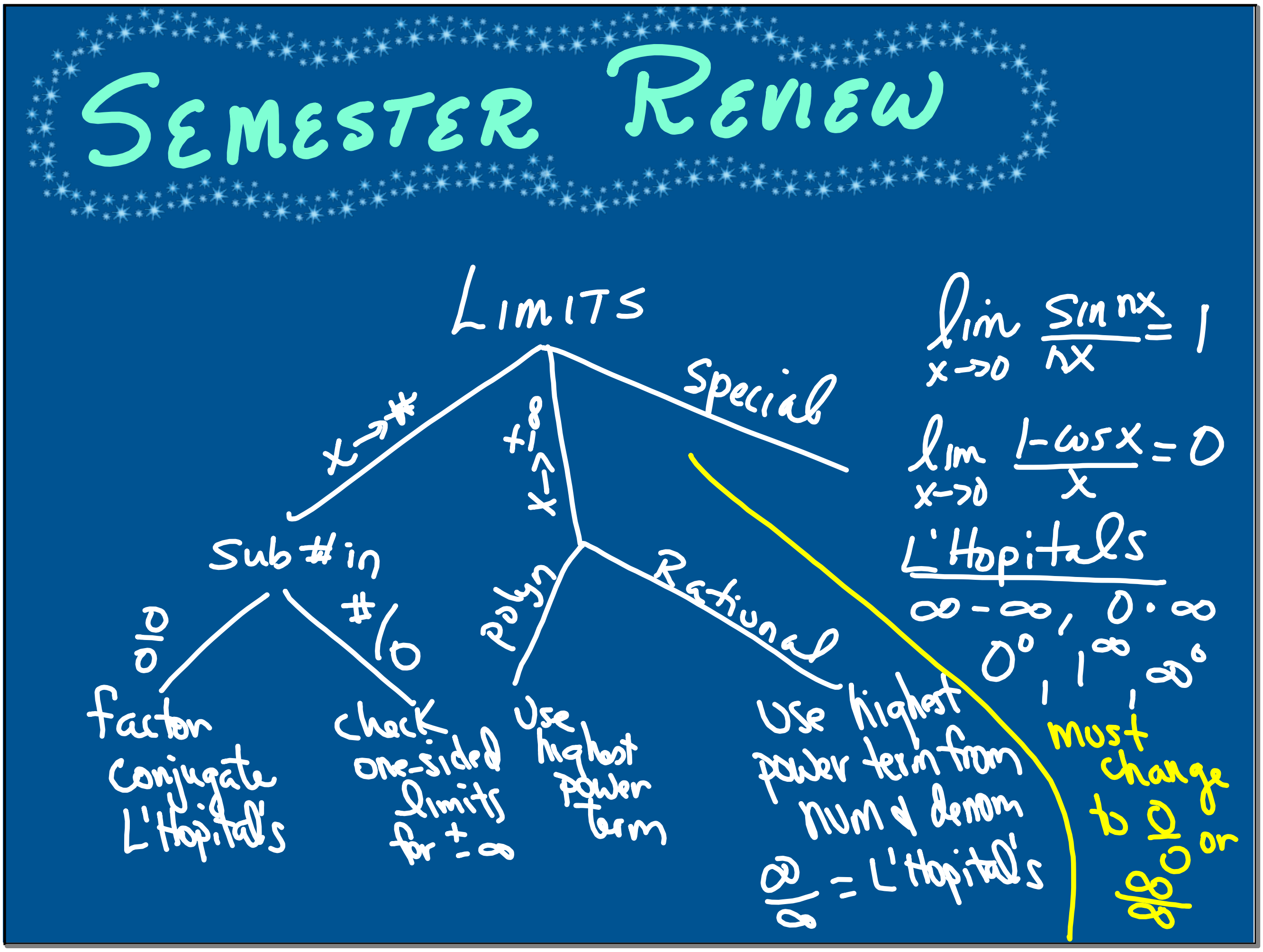
December 5, 2023



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$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}\cdot\frac{(\sqrt{x}+3)}{(\sqrt{x}+3)} = \frac{0}{0}$$

$$\lim_{x\to 9} \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} = \frac{1}{6}$$

L'Hop.

$$\lim_{x\to 9} \frac{\frac{1}{2}x^{-1/2}}{1}$$

$$\lim_{x\to 9} \frac{1}{2\sqrt{x}} = \frac{1}{6}$$

$$\lim_{x\to 3} \frac{4-x}{x^2-9} = \frac{1}{0} = \text{DNE}$$

$$\lim_{x\to 3^-} \frac{4-x}{x^2-9} = \frac{+}{-} = -\infty$$

$$\lim_{x\to 3^+} \frac{4-x}{x^2-9} = \frac{+}{+} = +\infty$$

$$\lim_{x\to -\infty} \frac{\sqrt{4x^2-3x+1}}{6x-5}$$

$$\lim_{x\to -\infty} \frac{\sqrt[2]{4x^2}}{6x}$$

$$\lim_{x\to -\infty} \frac{2|x|}{6x}$$

$$\lim_{x\to -\infty} -\frac{2\cancel{x}}{6\cancel{x}} = -\frac{1}{3}$$

$$\lim_{x\to 0} x \cot x = 0\cdot\infty$$

$$\lim_{x\to 0} \frac{x}{\tan x}$$

$$\lim_{x\to 0} \frac{1}{\sec^2 x} = \frac{1}{1} = 1$$

$$\lim_{x\to 0} \frac{2\cdot\sin 8x}{2\cdot 4x} \qquad \frac{\cos 8x\cdot 8}{4} = \frac{8}{4} = 2$$

$$2\lim_{x\to 0} \frac{\sin 8x}{8x}$$

$$2\cdot 1 = 2$$

$$\lim_{x\to 0} x^{2x^2} = 0^0$$

$$\lim_{x\to 0} e^{\ln x^{2x^2}}$$

$$\lim_{x\to 0} e^{2x^2\cdot\ln x}$$

$$\lim_{x\to 0^+} 2x^2\cdot\ln x$$

$$\lim_{x\to 0^+} \frac{2\ln x}{x^{-2}} = \frac{-\infty}{\infty}$$

$$\frac{1}{x^2}$$

$$\lim_{x\to 0} \frac{2\cdot\frac{1}{x}}{-\frac{2}{x^3}}$$

$$\lim_{x\to 0} \frac{\cancel{2}}{\cancel{x}}\cdot\frac{x^{\cancel{3}\,2}}{\cancel{-2}}$$

$$= 0$$

$$\boxed{e^0 = 1}$$

$$\lim_{x\to -\infty} e^x = 0 \qquad \lim_{x\to 0^+} \ln x = -\infty$$

$$\lim_{x\to \infty} e^x = \infty \qquad \lim_{x\to\infty} \ln x = +\infty$$

$$f(x)=\begin{cases} x^3+2x+1 & x\le 2 \\ 5x+3 & x>2 \end{cases} \quad a=2$$

1) $f(2)=8+4+1=13$

2) $\lim_{x\to 2^-} x^3+2x+1=13$

$\lim_{x\to 2^+} 5x+3=13$

$\lim_{x\to 2} f(x)=13$

3) $f(2)=\lim_{x\to 2} f(x)$

Yes. f is continuous

4) $f'(2)^-=3x^2+2=14$

$f'(2)^+=5$

$f'(2)^-\neq f'(2)^+$

not diff.

$$\text{continuity}\begin{cases} \text{1) } f(a) \text{ is defined.} \\ \text{2) } \lim_{x\to a} f(x) \text{ exists} \\ \text{3) } f(a)=\lim_{x\to a} f(x) \end{cases}$$

$$\text{differentiable}\begin{cases} \text{1) – 3)} \\ \text{4) } f'(a)^-=f'(a)^+ \end{cases}$$

Definitions of Deriv.

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a} \qquad \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$f(x) = x^2 - 3x + 1$$

$$\lim_{x \to a} \frac{x^2 - 3x + \cancel{1} + (-a^2 + 3a - \cancel{1})}{x-a}$$

$$\lim_{x \to a} \frac{(x^2 - a^2)(-3x + 3a)}{x-a}$$

$$\lim_{x \to a} \frac{\cancel{(x-a)}(x+a) - 3\cancel{(x-a)}}{\cancel{x-a}}$$

$$= (a+a) - 3$$

$$= 2a - 3$$

$$\lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) + 1 - (x^2 - 3x + 1)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h + \cancel{1} - \cancel{x^2} + \cancel{3x} - \cancel{1}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$$

$$= 2x - 3$$

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}a^x = \ln a \cdot a^x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$y = (\sin x)^{x^2}$$

$$y = e^{\ln(\sin x)^{x^2}} = e^{x^2 \cdot \ln(\sin x)}$$

$$y' = e^{x^2 \cdot \ln(\sin x)} \cdot \left[x^2 \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot 2x\right]$$

$$= (\sin x)^{x^2} \cdot \left[x^2 \cot x + 2x \ln(\sin x)\right]$$

$$= x(\sin x)^{x^2}\left[x \cot x + 2\ln(\sin x)\right]$$

$$f(x) = \csc^{-1}\left(\sqrt{x^2-1}\right) \qquad \frac{-1}{|x|\sqrt{x^2-1}}$$

$$f'(x) = \frac{-1}{\left|\sqrt{x^2-1}\right| \sqrt{\underbrace{\left(\sqrt{x^2-1}\right)^2 - 1}_{x^2-1-1}}} \cdot \frac{1}{\cancel{2}}(x^2-1)^{-1/2} \cdot \cancel{2}x$$

$$= \frac{-1}{\sqrt{x^2-1}\,\sqrt{x^2-2}} \cdot \frac{x}{\sqrt{x^2-1}}$$

$$= \frac{-x}{(x^2-1)\sqrt{x^2-2}}$$