December 7, 2023

December 7, 2023

Semester Review Day 2

* Deriv of trig func + inverses, e^x, a^x, $\ln x$
* Both definitions of derivative – Don't forget limit!
* A derivative represents. . ..

Find equation of tangent line to $f(x) = 3x^2 - 4x + 1$ at $x = -2$.

slope $f'(x) = 6x - 4$

$f'(-2) = 6(-2) - 4 = -16$

point: $f(-2) = 3(-2)^2 - 4(-2) + 1$

$= 12 + 8 + 1$

$= 21$

$(-2, 21)$

$y - y_1 = m(x - x_1)$

$y - 21 = -16(x + 2)$

$y - 21 = -16x - 32$

$y = -16x - 11$

$(2x^4)(y^2) - 7^{x^6} = 4\sin x + 5y^8$ Find $\frac{dy}{dx}$

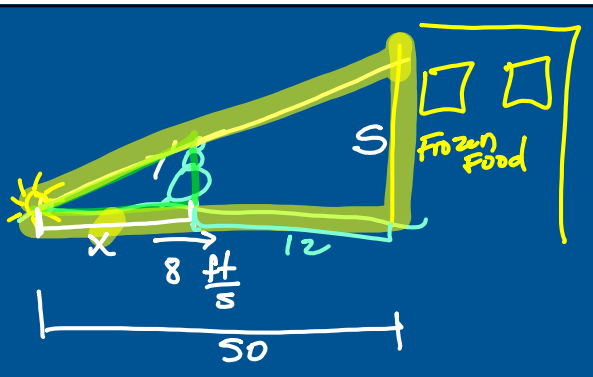
$2x^4 \cdot 2y\frac{dy}{dx} + y^2 \cdot 8x^3 - \ln 7 \cdot 7^{x^6} \cdot 6x^5 = 4\cos x + 40y^7\frac{dy}{dx}$

$4x^4 y\frac{dy}{dx} + 8x^3y^2 - 6\ln 7 \cdot 7^{x^6} \cdot x^5 = 4\cos x + 40y^7\frac{dy}{dx}$

$\frac{dy}{dx}(4x^4y - 40y^7) = 4\cos x + 6\ln 7 \cdot 7^{x^6}x^5 - 8x^3y^2$

$$\frac{dy}{dx} = \frac{4\cos x + 6\ln 7 \cdot 7^{x^6}x^5 - 8x^3y^2}{4x^4 - 40y^7}$$

$$= \frac{2\cos x + 3\ln 7 \cdot 7^{x^6} \cdot x^5 - 4x^3y^2}{2x^4 - 20y^7}$$



How fast is his shadow changing when he is 12 ft. from the building?

Light is 50' from bldg.

b $\frac{x}{7} = \frac{50}{S}$

h $\frac{38}{7} = \frac{50}{S}$

$\frac{d}{dt}\left[x \cdot S = 350\right]$

$385 = 350$

$S = \frac{350}{38} = \frac{175}{19}$

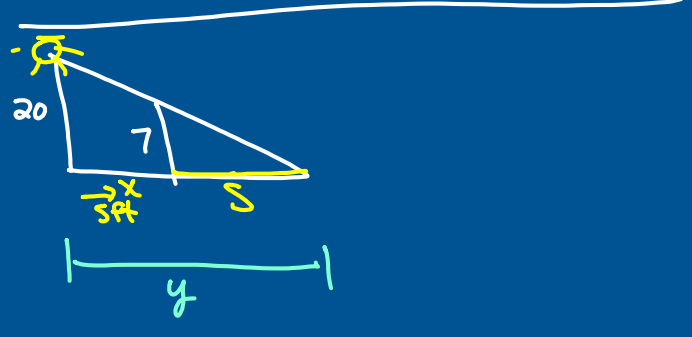
$x \cdot \frac{ds}{dt} + S \cdot \frac{dx}{dt} = 0$

$38 \cdot \frac{ds}{dt} + \frac{175}{19} \cdot 8 = 0$

$38\frac{ds}{dt} + \frac{1400}{19} =$

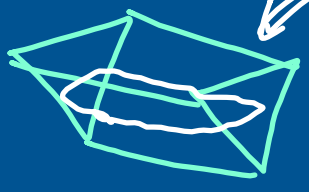
$\frac{1}{38} \cdot 38 \frac{ds}{dt} = -\frac{1400}{19} \cdot \frac{1}{38}$

$\frac{ds}{dt} = -1.94$



$y = S + x$

$\frac{dy}{dt} = \frac{ds}{dt} + \frac{dx}{dt}$



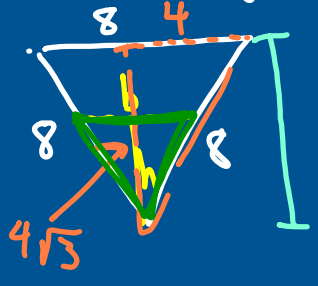
$20 \frac{ft^3}{sec}$

How fast is height changing? when $h = 5$ ft

Equilateral triangles

8 ft. sides

Length = 32 ft.



$$V = \frac{1}{2} b h \ell \qquad \ell = 32$$

$$V = 16bh$$

$$V = 16\left(\frac{2}{\sqrt{3}}h\right)h$$

$$\frac{h}{b} = \frac{4\sqrt{3}}{8}$$

$$8h = 4\sqrt{3}b$$

$$\frac{8h}{4\sqrt{3}} = b$$

$$\frac{2}{\sqrt{3}}h = b$$

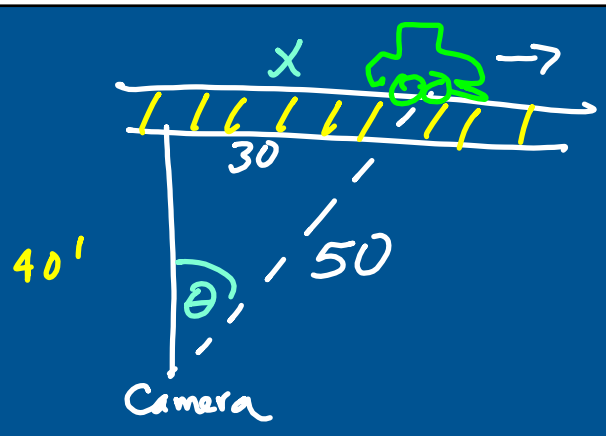
$$\frac{d}{dt}\left[V = \frac{32}{\sqrt{3}}h^2\right]$$

$$\frac{dV}{dt} = \frac{64}{\sqrt{3}}h\frac{dh}{dt}$$

$$20 = \frac{64}{\sqrt{3}}(5)\frac{dh}{dt}$$

$$\frac{\sqrt{3}}{320}\ 20 = \frac{320}{\sqrt{3}}\frac{dh}{dt}$$

$$\frac{\sqrt{3}}{16}\ ft/s = \frac{dh}{dt}$$



How fast is camera turning when distance from camera to train is 50 ft.?

$x^2 + 40^2 = 50^2$

$x = 30$

$\tan\theta = \frac{x}{40}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{40}\frac{dx}{dt}$

$\frac{\text{hyp}}{\text{adj}}\left(\frac{50}{40}\right)^2 \frac{d\theta}{dt} = \frac{1}{40}\frac{dx}{dt}$

$\frac{25}{16}\frac{d\theta}{dt} = \frac{1}{40}\cdot(44)$

$\frac{16}{25}\cdot\frac{25}{16}\frac{d\theta}{dt} = \frac{11}{10}\cdot\frac{16}{25}$

$\frac{d\theta}{dt} = \frac{88}{125}\ \frac{\text{rad}}{\text{s}}$