ANTIDIFFERENTIATION (Integration)

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$\int dy = \int f'(x)\,dx$

$y = \int f'(x)\,dx$

$\int (4x^1 + 9x^2)\,dx$

$\frac{4x^2}{2} \quad \frac{9x^3}{3}$

$= \boxed{2x^2 + 3x^3 + C}$

Indefinite Integrals

Power Rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \left(\frac{2}{x^3} + 4\sqrt[3]{x} - \frac{1}{x^{3/5}} + 7\right) dx$$

$$\int \left(2x^{-3} + 4x^{1/3} - x^{-3/5} + 7\right) dx$$

$$= \frac{2x^{-2}}{-2} + \frac{3}{\cancel{4}} \cdot 4x^{4/3} - \frac{5}{2}x^{2/5} + 7x + C$$

$$= \frac{-1}{x^2} + 3x^{4/3} - \frac{5}{2}x^{2/5} + 7x + C$$

$$\int (7x^2 - 4)^2 \, dx$$

$$(7x^2-4)(7x^2-4)$$

$$\int \left(49x^4 - 56x^2 + 16\right) dx$$

$$= \frac{49x^5}{5} - \frac{56x^3}{3} + 16x + C$$

$$\int \frac{4x^2-2x+1}{\sqrt{x}}\,dx$$

$$\int (4x^2-2x^1+1)\,x^{-1/2}\,dx$$

$$\int \left(4x^{3/2}-2x^{1/2}+x^{-1/2}\right)dx$$

$$= \frac{2}{5}\cdot 4x^{5/2} - \frac{2}{3}\cdot 2x^{3/2} + \frac{2}{1}x^{1/2} + C$$

$$= \frac{8}{5}x^{5/2} - \frac{4}{3}x^{3/2} + 2x^{1/2} + C$$

Initial value problems.

Find y.

$$\int \frac{dy}{\cancel{dx}} = \int (3x^2+2x)\,dx \qquad y(2)=7$$

$$y = \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + C$$

$$7 = (2)^3 + (2)^2 + C$$

$$7 = 8 + 4 + C$$

$$-12$$

$$-5 = C$$

$$\boxed{y = x^3 + x^2 - 5}$$

U-Substitution

$$\int 6x(x^2+5)^8\,dx$$

$u = x^2+5$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int \overset{3}{\cancel{6x}}\, u^8\, \frac{du}{\cancel{2x}}$$

$$\int 3u^8\,du$$

$$= \frac{3u^9}{9} + C$$

$$= \boxed{\frac{1}{3}(x^2+5)^9 + C}$$

$$\int \frac{3x}{\sqrt{4-3x^2}}\,dx$$

$u = 4-3x^2$

$du = -6x\,dx$

$\frac{du}{-6x} = dx$

$$\int 3x(4-3x^2)^{-1/2}\,dx$$

$$\int \cancel{3x}\cdot u^{-1/2}\cdot \frac{du}{\underset{-2}{-\cancel{6x}}}$$

$$-\frac{1}{2}\int u^{-1/2}\,du$$

$$-\frac{1}{\cancel{2}}\cdot\cancel{2}\,u^{1/2} + C$$

$$-u^{1/2} + C$$

$$-(4-3x^2)^{1/2} + C$$

$$\int 2x\,dx$$

$$= 2\int x\,dx$$

$$\frac{d}{dx}\sin x = \cos x \qquad \int \cos x\, dx = \sin x + C$$

$$\frac{d}{dx}\cos x = \underline{-\sin x} \qquad \int \sin x\, dx = -\cos x + C$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \int \sec^2 x\, dx = \tan x + C$$

$$\frac{d}{dx}\cot x = -\csc^2 x \qquad \int \csc^2 x\, dx = -\cot x + C$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \int \sec x \tan x\, dx = \sec x + C$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \qquad \int \csc x \cot x\, dx = -\csc x + C$$

$$\frac{d}{dx}e^x = e^x \qquad \int e^x\, dx = e^x + C$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \qquad \int \frac{1}{x}\, dx = \ln|x| + C$$

$$\cancel{\int x^{-1}} = \frac{x^0}{0}$$

$$\int (4\cos x - 3\sec^2 x + 4\csc\cot x)\,dx$$

$$= 4\sin x - 3\tan x - 4\csc x + C$$

$$\int \csc x\,(\csc x - \cot x)\,dx$$

$$\int (\csc^2 x - \csc x\cot x)\,dx$$

$$= -\cot x + \csc x + C$$

$$\int \left(\frac{1}{\sin^2 x} - \cot x \sin x\right) dx$$

$$\int \left(\csc^2 x - \frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}\right) dx$$

$$= -\cot x - \sin x + C$$

$$\int \left(\frac{6}{x} + 5e^x\right) dx$$

$$\int \left(6 \cdot \frac{1}{x} + 5e^x\right) dx$$

$$= 6\ln|x| + 5e^x + C$$