

MORE INTEGRATION

$$\int 5x \sqrt{2x+3} dx$$

$$u = 2x + 3$$

$$du = 2 dx$$

$$\frac{du}{2} = dx$$

$$u - 3 = 2x$$

$$\frac{u - 3}{2} = x$$

$$\int 5x \cdot u^{1/2} \cdot \frac{du}{2}$$

$$\frac{5}{2} \int \frac{u-3}{2} \cdot u^{1/2} \cdot du$$

$$\frac{5}{4} \int (u-3) u^{1/2} du$$

$$\frac{5}{4} \int (u^{3/2} - 3u^{1/2}) du$$

$$\frac{5}{4} \left[\frac{2}{5} u^{5/2} - \frac{2 \cdot 3}{3} u^{3/2} \right] + C$$

$$= \frac{1}{2} u^{5/2} - \frac{5}{2} u^{3/2} + C$$

$$= \frac{1}{2} (2x+3)^{5/2} - \frac{5}{2} (2x+3)^{3/2} + C$$

$$\int (x+2)^2 \sqrt{1+x} \, dx$$

$$u = 1+x \Rightarrow u-1 = x \\ du = dx$$

$$\int (u-1+2)^2 u^{1/2} \, du$$

$$\int (u+1)^2 \cdot u^{1/2} \, du$$

$$\int (u^2 + 2u + 1) \cdot u^{1/2} \, du$$

$$\int (u^{5/2} + 2u^{3/2} + u^{1/2}) \, du$$

$$= \frac{2}{7} u^{7/2} + \frac{2 \cdot 2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{7} (1+x)^{7/2} + \frac{4}{5} (1+x)^{5/2} + \frac{2}{3} (1+x)^{3/2} + C$$

$$\int \tan^8 x \sec^2 x \, dx$$

$$\int u^8 \cdot \frac{\sec^2 x \cdot du}{\sec^2 x}$$

$$= \frac{u^9}{9} + C$$

$$= \frac{\tan^9 x}{9} + C$$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\frac{du}{\sec^2 x} = dx$$

$$\int x^6 \sin(x^7) dx$$

$$\int x^6 \sin u \frac{du}{7x^6}$$

$$\frac{1}{7} \int \sin u du$$

$$= -\frac{1}{7} \cos u + C$$

$$= -\frac{1}{7} \cos(x^7) + C$$

$$u = x^7$$

$$du = 7x^6 dx$$

$$\frac{du}{7x^6} = dx$$

$$\begin{aligned} & \int \frac{1}{y^2} \sec\left(\frac{1}{y}\right) \tan\left(\frac{1}{y}\right) dy & u &= \frac{1}{y} = y^{-1} \\ & & du &= -\frac{1}{y^2} dy \\ & = \int \frac{1}{\cancel{y^2}} \sec u \tan u \cdot \cancel{-y^2} du & -y^2 \cdot du &= dy \\ & = - \int \sec u \tan u du \\ & = -\sec u + C \\ & = -\sec\left(\frac{1}{y}\right) + C \end{aligned}$$

$$\int \frac{8 \cos(4x-7)}{\sin^6(4x-7)} dx$$

$$\frac{2}{8} \int \frac{\cancel{\cos(4x-7)}}{u^6} \cdot \frac{du}{\cancel{4 \cos(4x-7)}} \left(\frac{du}{4 \cos(4x-7)} = dx \right)$$

$$2 \int \frac{1}{u^6} du$$

$$2 \int u^{-6} du$$

$$= \frac{2u^{-5}}{-5} + C$$

$$= \frac{-2}{5u^5} + C$$

$$= \boxed{\frac{-2}{5 \sin^5(4x-7)} + C}$$

$$u = \sin(4x-7)$$

$$du = \cos(4x-7) \cdot 4 dx$$

