RATIONAL EXPONENTS

RULE #6

$$a^{m/n} = \sqrt[n]{a^m}$$

$$x^{2/3} = \sqrt[3]{x^2}$$

$$\sqrt[4]{p^3} = p^{3/4}$$

$$\sqrt[7\cdot 2]{x^{2\cdot 2}y^{3\cdot 2}} \cdot \sqrt[2\cdot 7]{x^7y^7}$$

$$\sqrt[14]{x^4y^6} \cdot \sqrt[14]{x^7y^7}$$

$$= \sqrt[14]{x^{11}y^{13}}$$

Rational #'s = fractions

$$\sqrt[3\cdot 4]{a^{3\cdot 3}b^{3\cdot}} \cdot \sqrt[3\cdot 4]{a^{1\cdot 4}b^{2\cdot 4}} \qquad a^3 \cdot a^5$$

$$\cancel{a^{3/4}b^{1/4} \cdot a^{1/3}b^{2/3}}$$

$$\cancel{a^{9/12}b^{3/12} \cdot a^{4/12}b^{8/12}}$$

$$\sqrt[12]{a^9b^3} \cdot \sqrt[12]{a^4b^8}$$

$$= \sqrt[12]{a^{13}b^{11}}$$

$$= a\sqrt[12]{a^1b^{11}}$$

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EVALUATE. $\Longleftarrow$ Numerical answer

$8^{1/3} = \sqrt[3]{8^1} = 2$

$25^{3/2} = \sqrt[2]{25^3} = 5^3 = 125$

$16^{3/4} = \sqrt[4]{16^3} = 2^3 = 8$

$\dfrac{a^{3+4}}{\cancel{a^4}}$

$81^{-1/2} = \dfrac{1}{\sqrt[2]{81^1}} = \dfrac{1}{9}$

$32^{-2/5} = \dfrac{1}{\sqrt[5]{32^2}} = \dfrac{1}{2^2} = \dfrac{1}{4}$

$\left(\dfrac{49}{16}\right)^{-3/2} = \left(\dfrac{16}{49}\right)^{3/2} = \sqrt[2]{\left(\dfrac{16}{49}\right)^3} = \left(\dfrac{4}{7}\right)^3 = \dfrac{64}{343}$

Write as a single radical & simplify.

$$\sqrt[5]{\sqrt[3]{x}} = \left(x^{1/3}\right)^{1/5} = x^{1/15} = \sqrt[15]{x}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[m\cdot n]{a}$$

$$\sqrt[4]{\sqrt[3]{\sqrt[2]{x}}} = \sqrt[24]{x}$$

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SOLVE.

Quadratic Form.

$x^2 - 2x^1 - 3 = 0$

$(x - 3)(x + 1) = 0$

$x - 3 = 0 \quad x + 1 = 0$

$\boxed{x = 3 \quad x = -1}$

$\sqrt[3]{x^1} = 7$

Which of these is in quadratic form?

$x^6 - 2x^3 - 8 = 0$ yes

$x^{2/5} - 3x^{1/5} + 2 = 0$ yes

$x^8 + 3x^2 + 2 = 0$ No

* Middle power is half of power on 1st term
* factor using the power on the middle term.

$x^{2/3} - 3x^{1/3} - 28 = 0$

$(x^{1/3} - 7)(x^{1/3} + 4) = 0$

$x^{1/3} - 7 = 0 \quad x^{1/3} + 4 = 0$

$(x^{1/3})^3 = (7)^3 \quad (x^{1/3})^3 = (-4)^3$

$\boxed{x = 343 \quad x = -64}$

SOLVING RADICAL EQUATIONS

$5\sqrt[3]{x+7} - 10 = 5$
$\quad +10 \quad +10$

1) Isolate root
2) Take to power

$1^3 + 2^3 = 3^3$

$1 + 8 = 27$

$(1+2)^3 = (3)^3$

$\frac{\cancel{5}\cdot\sqrt[3]{x+7}}{\cancel{5}} = \frac{15}{5}$

$\left(\sqrt[3]{x+7}\right)^3 = 3^3$

$x+7 = 27$

$\boxed{x = 20}$

Check answers if both sides raised to an even power

ULTIMATE PROBLEM

$$\sqrt{2x-2} - \sqrt{3x-2} = -1$$

1) Isolate one root

$$\left(\sqrt{2x-2}\right)^2 = \left(\sqrt{3x-2} - 1\right)^2$$

2) Square both sides

$$2x-2 = \left(\sqrt{3x-2}-1\right)\left(\sqrt{3x-2}-1\right) \longleftarrow \text{3) FOIL!}$$

$$2x-2 = 3x-\underline{2} - 1\sqrt{3x-2} - 1\sqrt{3x-2} \underline{+1}$$

$$2x-2 = 3x-1-2\sqrt{3x-2}$$
$$\quad\; -2x+2$$

4) Clean up like terms

$$\left(2\sqrt{3x-2}\right)^2 = (x+1)^2 \longleftarrow \text{FOIL!}$$

5) Isolate remaining root

6) Square both sides

$$4(3x-2) = (x+1)(x+1)$$

$$12x-8 = x^2+x+x+1$$
$$-12x+8 \qquad -12x+8$$

$$0 = x^2-10x+9$$

$$0 = (x-1)(x-9)$$

7) Set = 0, Factor, + Solve

$$x-1=0 \qquad x-9=0$$

$$\boxed{x=1 \qquad x=9}$$

8) Check solutions!

Check:

$$\sqrt{2x-2} - \sqrt{3x-2} = -1$$

$x=1$: $\sqrt{2-2} - \sqrt{3-2} = -1$

$$\sqrt{0} - \sqrt{1} = -1$$

$$0 - 1 = -1 \; ✓$$

$x=9$: $\sqrt{18-2} - \sqrt{27-2} = -1$

$$\sqrt{16} - \sqrt{25} = -1$$

$$4-5 = -1$$

$$-1 = -1 \; ✓$$