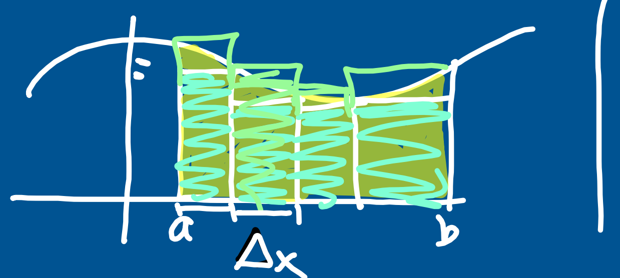
February 5, 2024

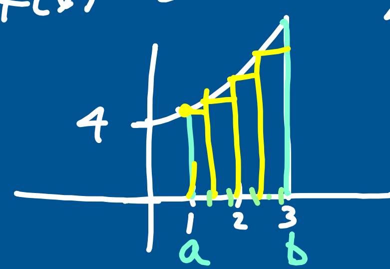
Riemann Sums



$f(x) = 3x^2 + 4$

$\overset{a}{[1}, \overset{b}{3]}$

4 sub-intervals



$\frac{3-1}{4} = \frac{b-a}{n} = \frac{1}{2}$

Left-hand sum

$\underset{w \cdot l}{\frac{1}{2}f(1)} + \underset{w \cdot l}{\frac{1}{2}f(1.5)} + \underset{w}{\frac{1}{2}}f(2) + \frac{1}{2}f(2.5)$

$\frac{1}{2}[f(1) + f(1.5) + f(2) + f(2.5)]$

$\frac{1}{2}[7 + 10.75 + 16 + 22.75]$

$= \frac{1}{2}[56.5]$

$= 28.25$ units^2

Right

$\frac{1}{2}[f(1.5) + f(2) + f(2.5) + f(3)]$

$\frac{1}{2}[10.75 + 16 + 22.75 + 31]$

$\frac{1}{2}[80.5]$

$= 40.25$ units^2

DEFINITE INTEGRALS

gives a numerical value

Indefinite
function + C

$$\int_{-2}^{5} (4x+3)\,dx$$

$$= \frac{2\cancel{4}x^2}{\cancel{2}} + 3x + C \Big|_{-2}^{5}$$

$$= 2(5)^2 + 3(5) + C - \left[2(-2)^2 + 3(-2) + C\right]$$

$$= 50 + 15 + \cancel{C} + \left[-8 + 6 + \cancel{C}\right] = \boxed{63}$$

$$\int_1^6 x\sqrt{x+3}\,dx$$

$u = x+3$ $\quad u-3 = x$
$du = dx$
$u = 1+3 = 4$
$u = 6+3 = 9$

$$\int_4^9 (u-3)\,u^{1/2}\cdot du$$

* change the limits of integration when you change to u's.
Do not put x's back in!

$$\int_4^9 \left(u^{3/2} - 3u^{1/2}\right)du$$

$$\frac{2}{5}u^{5/2} - \frac{\cancel{3}\cdot 2}{\cancel{3}}u^{3/2} \Big|_4^9$$

$$\frac{2}{5}\underset{\sqrt[2]{9^5}}{u^{5/2}} - 2\underset{\sqrt[2]{9^3}}{u^{3/2}} \Big|_4^9$$

$\sqrt[2]{4^5}$ $\quad \sqrt{4^3}$

$$\frac{2}{5}(243) - 2(27) + \left[-\frac{2}{5}(32) + 2(8)\right]$$

$$\frac{486}{5} - 54 - \frac{64}{5} + 16$$

$$= \frac{422}{5} - 38$$

$$= \frac{422}{5} - \frac{190}{5}$$

$$= \frac{232}{5}$$

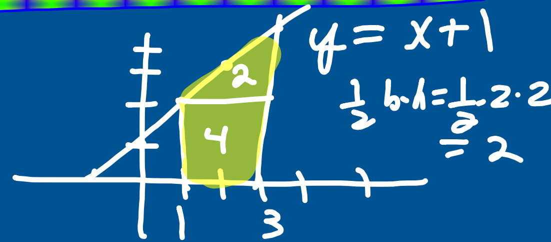
Fundamental Theorem of Calculus

Part 1

$$\int_1^3 (x+1)\,dx$$

$$= \frac{x^2}{2}+x\Big|_1^3$$

$$= \frac{9}{2}+3-\left(\frac{1}{2}+1\right) = 6$$



Area = 6

Integration represents the area between a curve and an axis.

$$\lim_{\Delta x \to 0} \sum_{x=a}^{b} \overset{h\cdot b}{f(x)\cdot \Delta x} = \int_a^b f(x)\,dx$$

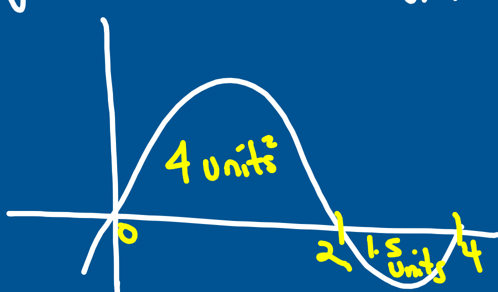
Let's rectangle width go to 0

Sum of rectangle

height of rectangle

base of rectangle

= Area of rectangle



$$\int_0^2 f(x)\,dx = 4 \text{ units}^2$$

$$\int_0^4 f(x)\,dx = 5.5 \text{ units}^2$$

$$\int_2^0 f(x)\,dx = -\int_0^2 f(x)\,dx$$

$$= -4$$

Part 2

$$\frac{d}{dx}\int_1^{x}(4t^2+t)\,dt = \frac{4t^3}{3}+\frac{t^2}{2}\Bigg|_1^{x^2+1}$$

$$\frac{d}{dx}\left[\frac{4}{3}\overset{(x^2+1)^3}{x^3}+\frac{\overset{(x^2+1)^2}{x^2}}{2}-\left(\frac{4}{3}+\frac{1}{2}\right)\right] = 4x^2+x$$

$$\frac{d}{dx}\int_6^{x}\frac{\sin^8(3t^2-1)}{\ln 8t^4}\,dt = \frac{\sin^8(3x^2-1)}{\ln 8x^4}$$

$$\frac{d}{dx}\int_2^{x^4}\frac{4}{\sqrt{t^3+2}}\,dt = \frac{4}{\sqrt{(x^4)^3+2}}\cdot 4x^3 = \frac{16x^3}{\sqrt{x^{12}+2}}$$

$$\frac{d}{dx}\int_{x^4}^{3x^7}\frac{2t}{t+1}\,dt$$

$$\frac{2(3x^7)}{3x^7+1}\cdot 21x^6 - \frac{2x^4}{x^4+1}\cdot 4x^3$$

$$\frac{d}{dx}\int_{\#}^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\cdot h'(x) - f(g(x))\cdot g'(x)$$