FUNDAMENTAL IDENTITIES

Identity – true for any ~~value~~ angle $\quad 2(x+5) = 2x+10$

Purpose: to simplify complicated expressions.

Reciprocal

1) $\csc\theta = \dfrac{1}{\sin\theta}$

$\sin\theta = \dfrac{1}{\csc\theta}$

2) $\sec\theta = \dfrac{1}{\cos\theta}$

3) $\cot\theta = \dfrac{1}{\tan\theta}$

Ratio

4) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

5) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean

6) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

7) $1 + \tan^2\theta = \sec^2\theta$

8) $1 + \cot^2\theta = \csc^2\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

Simplify.

$$\frac{\cos x}{\sin x}\frac{(1+\cos x)}{(1+\cos x)} + \frac{\sin x}{1+\cos x}\frac{(\sin x)}{(\sin x)}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{\sin x(1+\cos x)}$$

$$= \frac{\cos x + 1}{\sin x(1+\cos x)}$$

$$= \frac{1}{\sin x} \text{ OR } \csc x$$

$$(1+\tan x)^2 - 2\tan x$$

$$(1+\tan x)(1+\tan x) - 2\tan x$$

$$1 + 2\tan x + \tan^2 x - 2\tan x$$

($\tan x + \tan x$)

$$= \sec^2 x$$

Factor

$15\cos^2 x + 2\cos x - 8$

$15x^2 + 2x - 8$

$(5\cos x + 4)(3\cos x - 2)$

$\cos x = 2/3$

$$\frac{\sec^3 x - 8}{\sec^2 x - 4} \qquad \frac{x^3 - 8}{x^2 - 4}$$

$(x+2)(x-2)$

$$\frac{(\sec x - 2)(\sec^2 x + 2\sec x + 4)}{(\sec x + 2)(\sec x - 2)}$$

$$\frac{\sec^2 x + 2\sec x + 4}{\text{Se}}$$

Verify.

$x \cdot -y = -xy$

$$\tan^2(\theta)\left(\frac{1}{\sec^2\theta}\right) - \cot\theta \cdot \tan(\theta) = -\cos^2\theta$$

$$\left(\frac{\sin^2\theta}{\cos^2\theta}\right)(\cos^2\theta) - \left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right) = -\cos^2\theta$$

$$\sin^2\theta - 1 \qquad -\cos^2\theta$$

$$-\cos^2 x = -\cos^2 x$$

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$$\frac{\csc\theta \cdot \sec\theta}{\csc\theta \sin\theta} - \frac{\sec\theta \, \sin\theta}{\csc(+\theta)\, \sin\theta} = \frac{1}{\tan\theta}$$

$$\frac{\csc\theta\sec\theta - \sec\theta\sin\theta}{\sin\theta\csc\theta} = \cot x$$

$$\frac{\sec\theta(\csc\theta - \sin\theta)}{\sin\theta \cdot \frac{1}{\sin\theta}} = \frac{\cos x}{\sin x}$$

$$\frac{1}{\cos\theta}\left(\frac{1}{\sin\theta} - \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\sin\theta}\right) =$$

$$\frac{1}{\cos\theta}\left(\frac{1-\sin^2\theta}{\sin\theta}\right) = \frac{\cos\theta}{\sin\theta}$$

$$\frac{1}{\cos\theta}\left(\frac{\cos^2\theta}{\sin\theta}\right)$$

$$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\cot^4 B - \csc^4 B = 1 - 2\csc^2 B$$ (Power > 2, Factor!)

$$(\cot^2 B + \csc^2 B)(\cot^2 B - \csc^2 B) = 1 - 2\csc^2 B$$

$$(\cot^2 B + \csc^2 B)(-1)$$

$$-\cot^2 B - \csc^2 B = 1 - 2\csc^2 B$$

$$1 - \csc^2 B - \csc^2 B = 1 - 2\csc^2 B$$

$$1 - 2\csc^2 B = 1 - 2\csc^2 B$$

$$\frac{\cos^2 x + 3\sin x - 1}{3 + 2\sin x - \sin^2 x} = \frac{1}{1 + \csc x}$$

$3\sin x - \sin^2 x$

$$\frac{\cancel{1} - \sin^2 x + 3\sin x - \cancel{1}}{3 + 2\sin x - \sin^2 x} = \frac{1}{\frac{\sin x}{\sin x}\cdot 1 + \frac{1}{\sin x}}$$

$$\frac{\sin x\,\cancel{(3 - \sin x)}}{(1 + \sin x)\cancel{(3 - \sin x)}} = \frac{1}{\frac{\sin x + 1}{\sin x}}$$

← Keep-Change-Flip

$$= \frac{\sin x}{\sin x + 1}$$