INTEGRATION 3 – Integ with $e^x, a^x, \ln x, \log_b x$, inv trig func.

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int a^x\,dx = \frac{1}{\ln a}a^x + C$$

$$\frac{d}{dx}a^x = \ln a \cdot \int a^x$$

$$\int 7^{\sin x}\cdot \cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$\frac{du}{\cos x} = dx$

$$\int 7^u \cdot \cancel{\cos x}\cdot \frac{du}{\cancel{\cos x}}$$

$$= \frac{1}{\ln 7}\cdot 7^u + C$$

$$= \frac{1}{\ln 7}\cdot 7^{\sin x} + C$$

$$\int x \cdot e^{5x^2}\,dx \qquad u = 5x^2$$
$$du = 10x\,dx$$
$$\frac{du}{10x} = dx$$

$$\int \cancel{x} \cdot e^u \frac{du}{10\cancel{x}}$$

$$\frac{1}{10}\int e^u\,du$$

$$\frac{1}{10}e^u + C$$

$$\frac{1}{10}e^{5x^2} + C$$

$$\int \frac{e^{\tan y}}{\cos^2 y}\,dy$$

$$u = \tan y$$
$$du = \sec^2 y\,dy$$
$$\frac{du}{\sec^2 y} = dy$$

$$= \int \frac{e^u}{\cos^2 y}\,\frac{du}{\sec^2 y}$$

$$\int \frac{e^u}{\cos^2 y} \cdot \frac{du}{\frac{1}{\cos^2 y}}$$

$$\int \frac{e^u}{\cancel{\cos^2 y}} \cdot \cancel{\cos^2 y}\,du$$

$$= e^u + C$$

$$= \boxed{e^{\tan y} + C}$$

$$\int \frac{x^2}{1-x^3}\,dx \qquad u = 1-x^3$$
$$du = -3x^2\,dx$$
$$\frac{du}{-3x^2} = dx$$

$$\int \frac{\cancel{x^2}}{u}\cdot\frac{du}{-3\cancel{x^2}}$$

$$\cancel{\int u^{-1}\,du}$$

$$-\frac{1}{3}\int \frac{1}{u}\,du$$

$$-\frac{1}{3}\ln|u| + C$$

$$-\frac{1}{3}\ln|1-x^3| + C$$

$$\int \frac{\csc^2 x}{\cot x}\,dx \qquad u = \cot x$$
$$du = -\csc^2 x\,dx$$

$$\int \frac{\cancel{\csc^2 x}}{u}\cdot\frac{du}{-\cancel{\csc^2 x}}$$

$$-\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= -\ln|\cot x| + C$$

$$\int \frac{(\ln x)^5}{x}\,dx \qquad u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$\int \frac{u^5}{\cancel{x}} \cdot \cancel{x}\,du \qquad x\,du = dx$$

$$\int u^5\,du$$

$$= \frac{u^6}{6} + C$$

$$= \frac{(\ln x)^6}{6} + C$$

Inverse Trig Functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1} \qquad \int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C$$

$$\int \frac{6x^2}{\sqrt{1-\underbrace{25x^6}_{(5x^3)^2}}}\,dx \qquad u = 5x^3$$

$$du = 15x^2\,dx$$

$$\frac{du}{15x^2} = dx$$

$$\int \frac{6\cancel{x^2}}{\sqrt{1-u^2}}\,\frac{du}{15\cancel{x^2}}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$\frac{2}{5}\sin^{-1}u + C$$

$$2/5\,\sin^{-1}(5x^3) + C$$

$$\int \frac{3x}{4+9x^4}\,dx$$

1) Make the "1"

2) u-sub to get u^2

3) Integrate with inv trig func

$$\frac{3}{4}\int \frac{x}{1+\frac{9}{4}x^4}\,dx$$

$\left(\frac{3}{2}x^2\right)^2$

$u=\frac{3}{2}x^2$

$du=3x\,dx$

$\frac{du}{3x}=dx$

$$\frac{\cancel{3}}{4}\int \frac{\cancel{x}}{1+u^2}\,\frac{du}{\cancel{3x}}$$

$$\frac{1}{4}\int \frac{1}{1+u^2}\,du$$

$$\frac{1}{4}\tan^{-1}u + C$$

$$\frac{1}{4}\tan^{-1}\left(\frac{3}{2}x^2\right)+C$$

$$\int \frac{3x}{\sqrt{4-9x^4}}\,dx$$

$$\frac{1}{2}\int \frac{3x}{\sqrt{1-\frac{9}{4}x^4}}$$

$$\int \frac{4\cos x}{\sin x\sqrt{\sin^2 x-36}}\,dx$$

$$\frac{4}{6}\int \frac{\cos x}{\sin x\sqrt{\frac{\sin^2 x}{36}-1}}\,dx$$

$$\left(\frac{\sin x}{6}\right)^2$$

$u = \frac{1}{6}\sin x$ $\quad 6u = \sin x$

$$\frac{du}{\frac{1}{6}\cos x} = \frac{1}{6}\cos x\,dx$$

$$\frac{6}{\cos x}du = dx$$

$$\frac{2}{3}\int \frac{\cancel{\cos x}}{\cancel{6}u\sqrt{u^2-1}}\cdot \frac{\cancel{6}}{\cancel{\cos x}}du$$

$$\frac{2}{3}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$\frac{2}{3}\sec^{-1}u + C$$

$$2/3\sec^{-1}\left(\frac{\sin x}{6}\right) + C$$