

PHASE SHIFT & VERTICAL SHIFT

(Horizontal)

$$y = x^2$$

$$y = (x-5)^2 + 2$$

Right 5, Up 2

$$y = \sin\left(x + \frac{\pi}{4}\right) - 3$$

Left $\frac{\pi}{4}$ Down 3

$$y = \sin\left(2\left(x + \frac{\pi}{4}\right)\right) - 3$$

$$y = \sin\left(2x + \frac{\pi}{2}\right) - 3$$

$$2x + \frac{\pi}{2} = 0$$

$$\frac{1}{2} \cdot 2x = -\frac{\pi}{2} \cdot \frac{1}{2}$$

$$x = -\frac{\pi}{4}$$

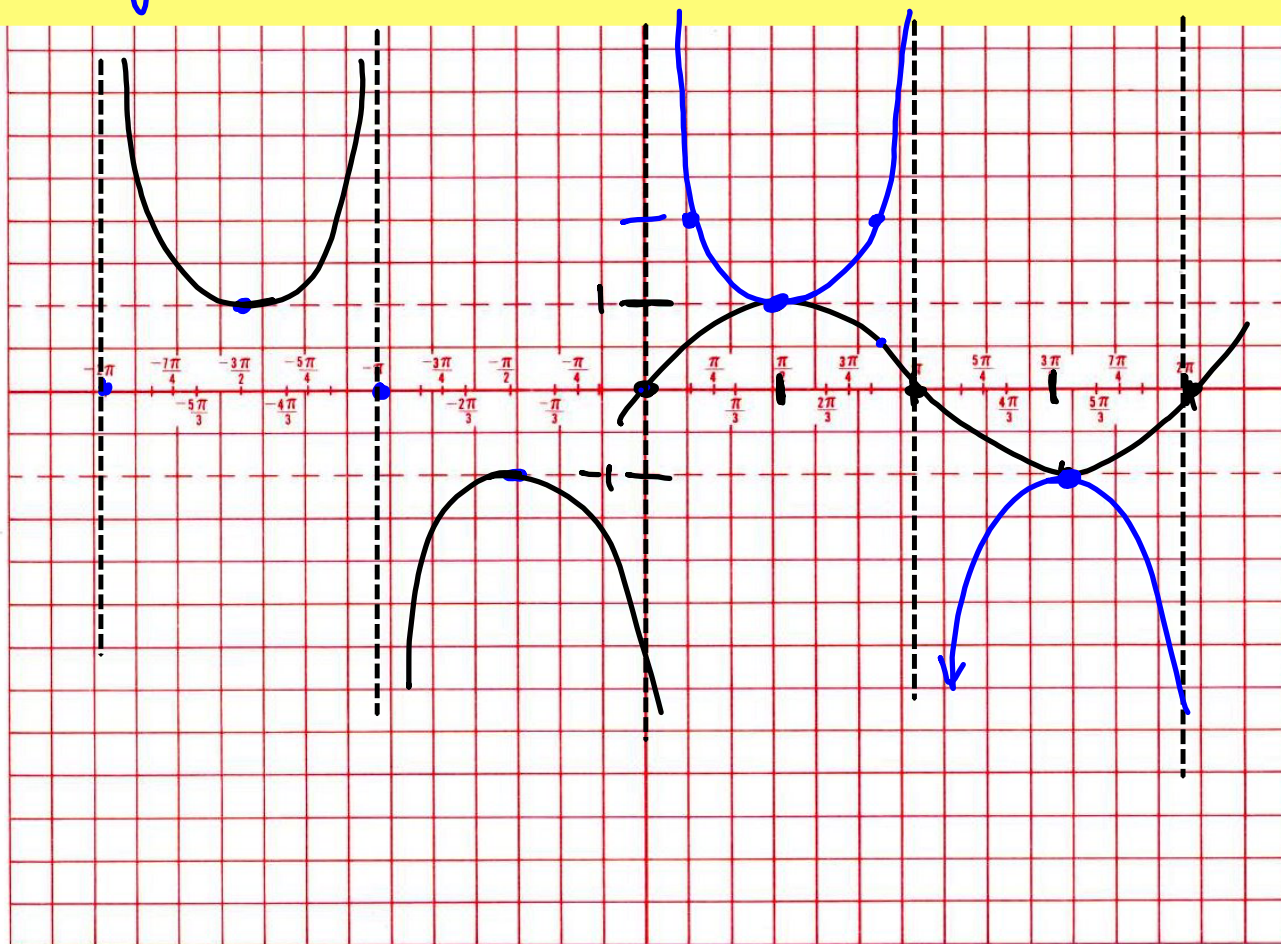
With + little $\sec x$ + $\csc x$

$$y = a \sin(bx+c) + d$$

	sin	csc
	cos	sec
amp	a	NA
period	$\frac{2\pi}{b}$	$\frac{2\pi}{b}$
vertical shift	d	d
phase shift (horiz. shift)	$bx+c=0$ $x = -\frac{c}{b}$	$bx+c=0$ $x = -\frac{c}{b}$

$$y = \sin x$$

$$y = \csc x$$



$$y = -2 \sin(2x - \pi) - 1$$

amp 2(-)

period $\frac{2\pi}{2} = \pi$

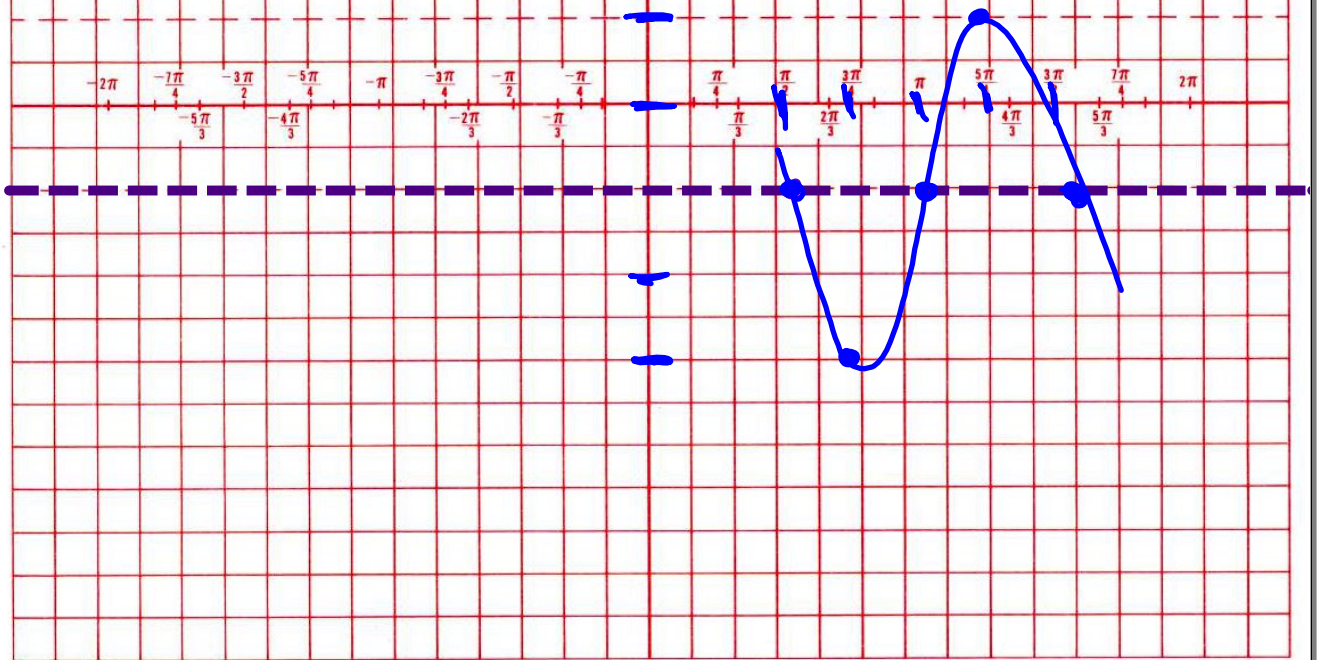
v.s. -1

p.s. $2x - \pi = 0$
 $x = \frac{\pi}{2}$

spacing
 period $\cdot \frac{1}{4}$

$\pi \cdot \frac{1}{4} = \frac{\pi}{4}$

$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
p.s.				
$\frac{2\pi}{4}$	$\frac{3\pi}{4}$	$\frac{4\pi}{4}$	$\frac{5\pi}{4}$	$\frac{6\pi}{4}$



$$y = 1 + 2 \sec\left(\frac{2}{3}x + \frac{7\pi}{9}\right)$$

v.s. 1
amp NA (2)

cos

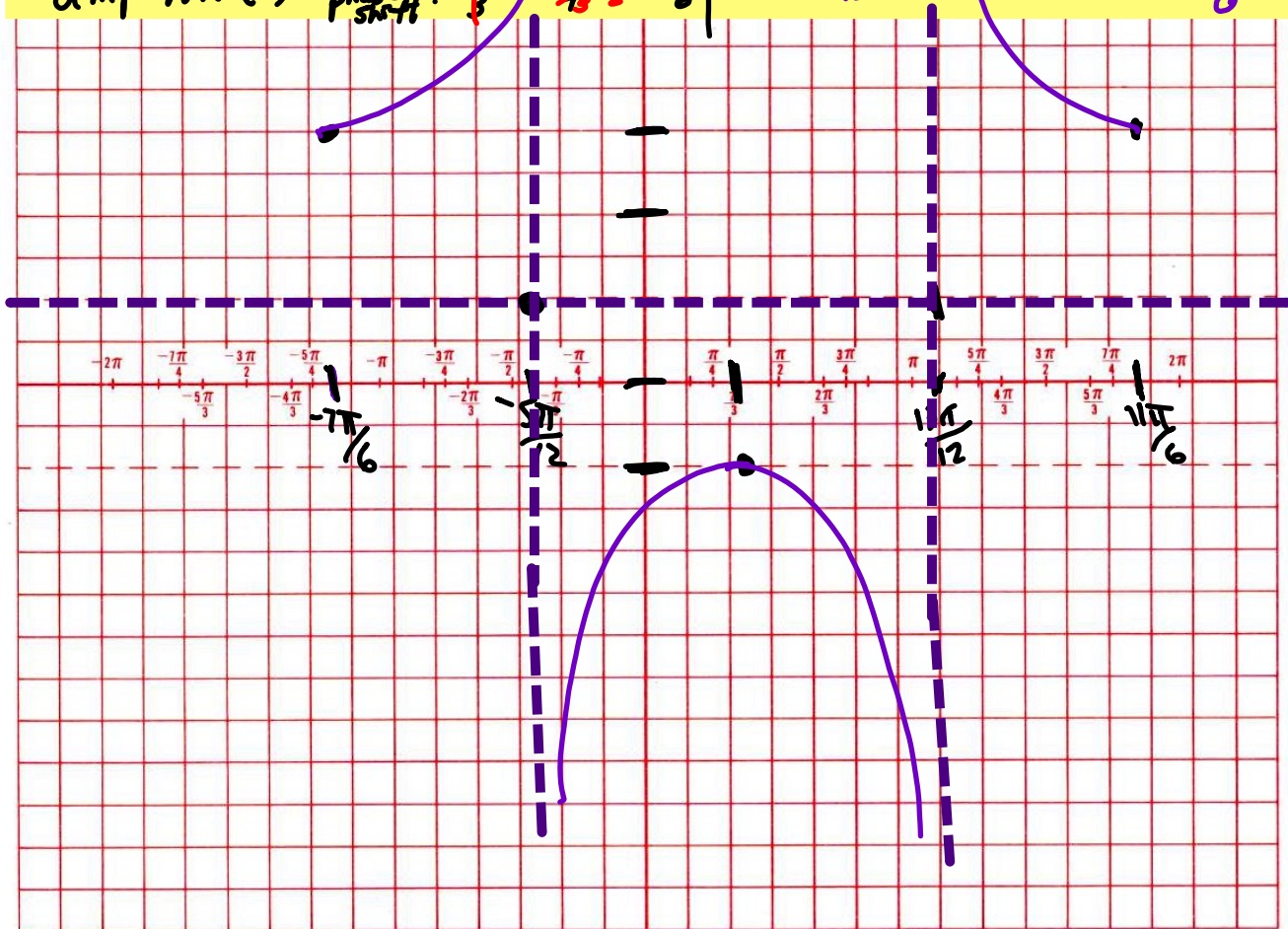
period: $\frac{2\pi}{2/3} = 2\pi \cdot \frac{3}{2} = 3\pi$

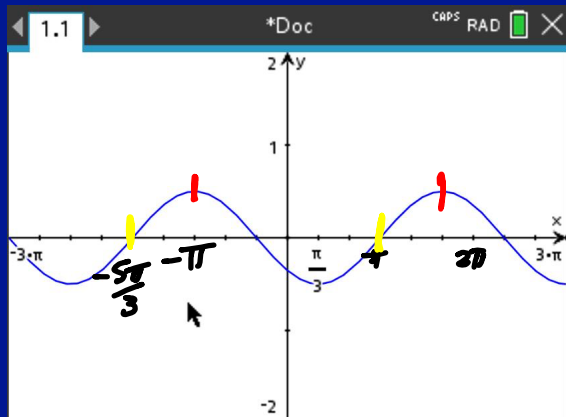
phase shift: $\frac{1}{3}x = \frac{7\pi}{9} \cdot \frac{3}{2} = \frac{7\pi}{6}$

spacing:

$$3\pi \cdot \frac{1}{4} = \frac{3\pi}{4} = \frac{9\pi}{12}$$

$-\frac{7\pi}{6}$	$-\frac{5\pi}{12}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{11\pi}{6}$
$-\frac{14\pi}{12}$	$-\frac{5\pi}{12}$	$\frac{4\pi}{12}$	$\frac{13\pi}{12}$	$\frac{23\pi}{12}$
		$\frac{\pi}{3}$		$\frac{11\pi}{6}$





$$y = \frac{1}{2} \cos\left(\frac{3}{4}(x + \pi)\right) - 1$$

$$y = \frac{1}{2} \cos\left(\frac{3}{4}x + \frac{3}{4}\pi\right) - 1$$

COS X

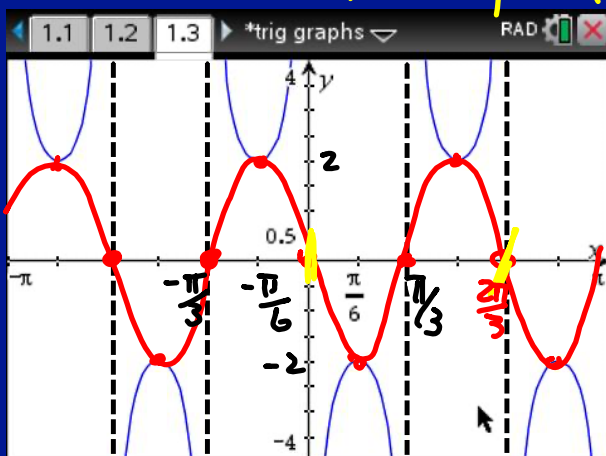
p.s. $-\pi$

amp. $1/2$

per. $\frac{3\pi}{3} + \frac{5\pi}{3} = \frac{8\pi}{3}$

V.S. -1

$$b = \frac{2\pi}{\frac{8\pi}{3}} = \frac{2\pi}{\frac{8\pi}{3}} = 2\pi \cdot \frac{3}{8\pi} = \frac{3}{4}$$



$$\frac{\pi}{3} + \frac{\pi}{3}$$

CSC X

V.S. 0

amp NA (2) (-1)

p.s. 0

period $\frac{2\pi}{3}$

p.s. $\frac{\pi}{3}$

$$b = \frac{2\pi}{\frac{2\pi}{3}} = 2\pi \cdot \frac{3}{2\pi}$$

$$y = -2 \csc(3x)$$

$$y = 2 \csc\left(3\left(x - \frac{\pi}{3}\right)\right)$$