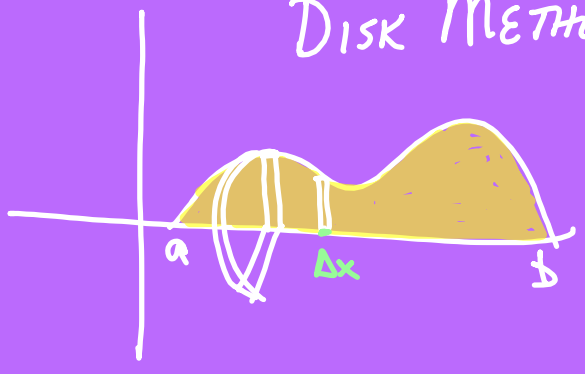
February 20, 2024

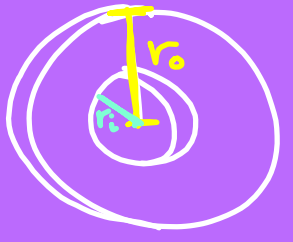
VOLUMES OF SOLIDS OF REVOLUTION

DISK METHOD



$$V = \pi r^2 h$$

$$\pi \int_a^b [f(x)]^2 \, dx$$



$$\pi r_o^2 h - \pi r_i^2 h$$

$$\pi \int_a^b \left(r_o^2 - r_i^2 \right) dx$$

$\square$ is $\perp$ to axis of revolution

Revolve region bounded by $y = 2x$ and $y = x^2$ around x-axis.

$$\pi \int_a^b \left(r_o^2 - r_i^2\right) dx$$

$$\pi \int_0^2 \left[(2x)^2 - (x^2)^2\right] dx$$

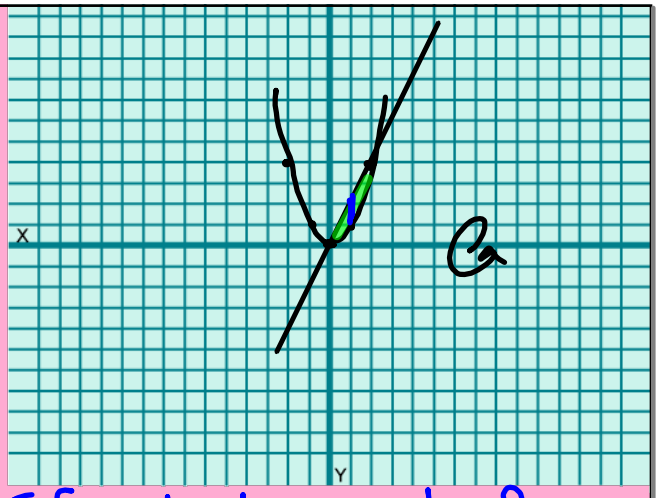
$$\pi \int_0^2 \left(4x^2 - x^4\right) dx$$

$$\pi \left[\frac{4}{3}x^3 - \frac{1}{5}x^5 \right]_0^2$$

$$\pi \left[\frac{32}{3} - \frac{32}{5} - (0-0) \right]$$

$$\frac{160}{15} - \frac{96}{15}$$

$$= \frac{64\pi}{15} \text{ units}^3$$



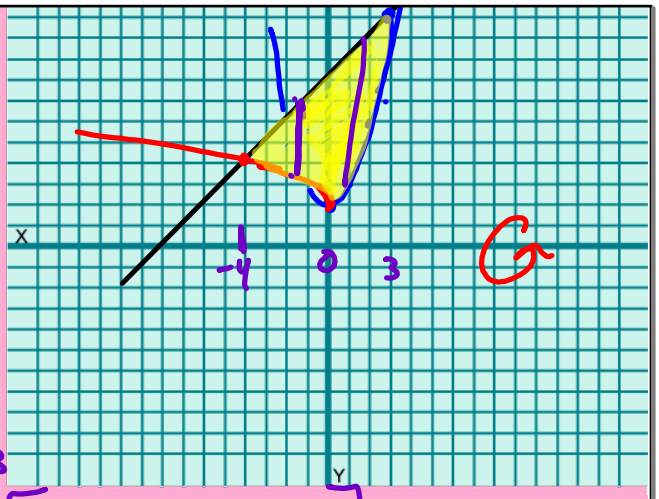
If rectangle is vertical,
$y = x$'s

If rectangle is horizontal
$x = y$'s

$y = x^2 + 2$ $\quad y = \sqrt{-x} + 2$

$y = x + 8$ $\quad$ around x-axis

0	0
−1	1
−4	2
−9	3



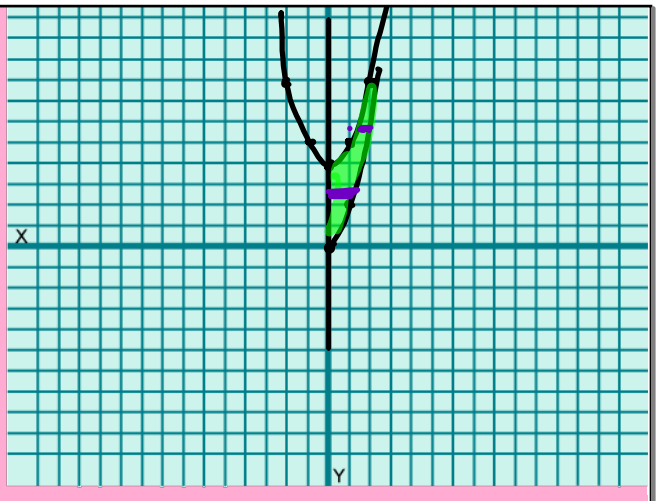
$$\pi \int_a^b \left(r_o^2 - r_i^2\right) dx$$

$$\pi \int_{-4}^{0} \left[(x+8)^2 - (\sqrt{-x}+2)^2\right] dx + \pi \int_{0}^{3} \left[(x+8)^2 - (x^2+2)^2\right] dx$$

$y = x^2 + 4 \quad y = 2x^2$

$x = 0$ around y-axis

0	0
1	2
2	8
3	18



$\sqrt{y-4} = \sqrt{x^2}$

$\sqrt{y-4} = x$

$y = 2x^2$

$\sqrt{\frac{y}{2}} = \sqrt{x^2}$

$$\pi \int_4^8 \left(\sqrt{\frac{y}{2}}\right)^2 - \left(\sqrt{y-4}\right)^2 dy$$

$$+ \pi \int_0^4 \left(\left(\sqrt{\frac{y}{2}}\right)^2 - 0^2\right) dy$$

$= 8\pi$ units^3