Combinatorics & Probability

of ways to perform an event

Tacos

Enchiladas

Rice

Beans

Funyuns
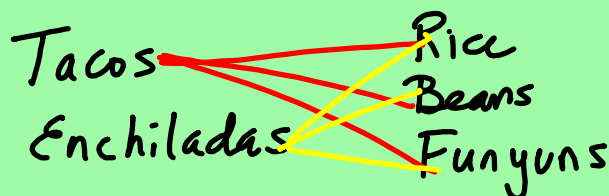
$2 \cdot 3 = 6$ meals

How many different meals are possible?

Fundamental Counting Principle--If there are p ways to do one event and q ways to do another, then there are p •q ways to do both.

Car manufacturer

8 body colors

2 fabrics

3 option packages

How many different cars can be made?

$8 \cdot 2 \cdot 3 = 48$

Permutations — # of arrangements (patterns) that can be made from a set of objects

Linear Permutations

$$7 \quad 125$$
$$25$$

1) All Objects $= n!$

How many ways are there to arrange the students sitting in the front row?

$$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 120$$

Factorial: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

2) Arrange a small group chosen from a larger group

15 students
5 desks

$$15 \cdot 14 \cdot 13 \cdot 12 \cdot 11$$

$$_nP_r = \frac{n!}{(n-r)!}$$

total # ↑   # to use

$$_{15}P_5 = \frac{15!}{(15-5)!} = \frac{15!}{10!}$$

$$\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdots}{10 \cdot 9 \cdot 6 \cdots}$$

How many ways can 4 relay runners be positioned for a race if chosen from 6 possible team members?

$$_6P_4 = 360$$

3) Alike Objects--<u>indistinguishable</u>, identical

How many permutations of the letters in the word MISSISSIPPI are possible?

$$\frac{11!}{4! \; 4! \; 2!} = 34{,}650$$
$$\quad\; S \;\; I \;\; P$$

$$\frac{total!}{alike! \; alike!}$$

How many ways can 5 identical basketballs, 4 identical volleyballs, and 3 identical soccer balls be arranged in a line?

$$\frac{12!}{5! \; 4! \; 3!} = 27{,}720$$

4) Specific locations or Repeated objects $=$ Draw blanks

— — — —

How many ways can six people from this class be arranged in row if there must be a sophomore on each end of the row and two juniors in the middle seats?

$$\underset{S_0}{\underline{11}} \cdot \underset{S_0}{\underline{11}} \cdot \underset{J}{\frac{4}{}} \cdot \underset{J}{\frac{3}{}} \cdot \underline{10} \cdot \underset{S_0}{\underline{10}} = 145,200$$

How many different license plates are possible with 3 letters followed by 3 digits, letters cannot be repeated but digits can be repeated?

$$\underline{26} \cdot \underline{25} \cdot \underline{24} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 15,600,000$$

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000$$

COMBINATIONS — the # of $\underline{groups}$ that can be formed from a set of objects

$$_n C_r = \frac{n!}{(n-r)! \; r!}$$

total → n ↑ # to use

$$_7 C_3 = \frac{7!}{4! \; 3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 35$$

Mrs. Meyer wants to form an "I Love Math" Committee to promote mathematics throughout the school. How many different committees of 5 students can be selected from this class?

$$_{15} C_5 = 3003$$

Card Facts

52 cards
4 suits - ♦ ♡ ♧ ♠
13 cards in a suit
4 cards of each type
12 face cards
26 red; 26 black

Draw 5 cards. How many hands of 5 diamonds are possible?

$$_{13}C_5 = 1287$$

How many hands with a full house are possible?

3 of a Kind
2 of a Kind

$$_{13}C_1 \cdot {}_4C_3 ; \quad _{12}C_1 \cdot {}_4C_2$$

$$= 3744$$

$$Prob = \frac{3744}{_{52}C_5} = \frac{3744}{2,598,960}$$

How many hands with 3 black cards and 2 red cards are possible?

$$_{26}C_3 \cdot {}_{26}C_2 = 845,000$$