APPLICATIONS OF INTEGRATION

Differential Equations

Find general solution:

$$\int \frac{d^2y}{dx^2} = \int 24x^2 + 18x + 4$$

$$\frac{dy}{dx} = \frac{24x^3}{3} + \frac{18x^2}{2} + 4x + C$$

$$\int \frac{dy}{dx} = \int 8x^3 + 9x^2 + 4x + C$$

$$y = \frac{8x^4}{4} + \frac{9x^3}{3} + \frac{4x^2}{2} + C_1x + C_2$$

$$\boxed{y = 2x^4 + 3x^3 + 2x^2 + C_1x + C_2}$$

Complete or general solution

+ C

Particular solution

Solve for C

Find particular solution.

$$\int \frac{d^2y}{dx^2} = \int 3x^2$$

$y = -1$ when $x = 0$

$y' = 9$ when $x = 2$

$$\frac{dy}{dx} = x^3 + C$$

$$9 = (2)^3 + C$$

$$-8$$

$$1 = C$$

$$\frac{dy}{dx} = x^3 + 1$$

$$\int \frac{dy}{dx} = \int x^3 + 1$$

$$y = \frac{x^4}{4} + x + C$$

$$-1 = 0 + 0 + C$$

$$\boxed{y = \frac{x^4}{4} + x - 1}$$

Motion

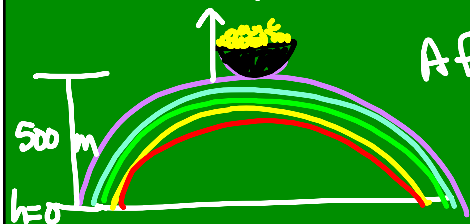
$s(t)$ = position

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

$a = -9.8 \text{ m/s}^2$

$a = -32 \text{ ft/s}^2$



After 2 sec, moving at 800 m/s

$\boxed{a(t) = -9.8}$

$v(t) = -9.8t + C$

$800 = -9.8(2) + C$

$819.6 = C$

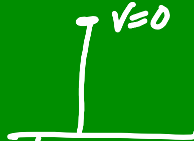
$\boxed{v(t) = -9.8t + 819.6}$

$s(t) = -4.9t^2 + 819.6t + C$

$500 = 0 + 0 + C$

$\boxed{s(t) = -4.9t^2 + 819.6t + 500}$

How high will it rise?



$0 = -9.8t + 819.6$

$83.6 = t$ sec

$s(83.6) = 34,772.7$ m

How fast will it be moving when it hits the ground

$0 = -4.9t^2 + 819.6t + 500$

quadr. formula

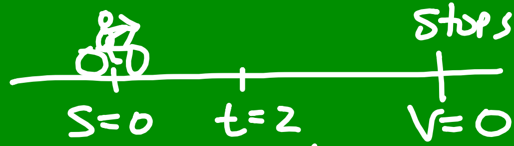
$t \approx 167.8$

solve(____ = 0, x)

$v = -9.8(167.8) + 819.6$

$v = -824.84$ m/s

A bicyclist applies his brakes & begins decelerating at 2 ft/s^2. How far will he travel before he comes to if his speed reduced to 6 ft/s after 2 sec?



$a(t) = -2$

$v(t) = -2t + C$

$6 = -2(2) + C$

$10 = C$

$\boxed{v(t) = -2t + 10}$

$\frac{-2t^2}{2}$

$s(t) = -t^2 + 10t + C$

$0 = 0 + 0 + C$

$\boxed{s(t) = -t^2 + 10t}$

$0 = -2t + 10$

$t = 5$ sec

How far will he travel?

$s(5) = -(5)^2 + 10(5)$

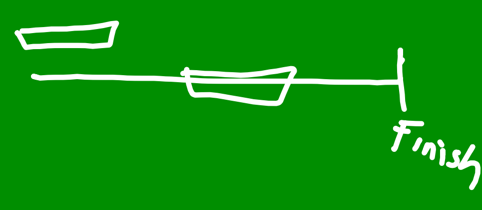
$= -25 + 50$

$= \boxed{25 \text{ ft}}$

Homework

a
v
s

a
v
s



BUSINESS APPLICATIONS

The marginal revenue for Apple watches is expressed by $\frac{dR}{dx} = 60,000 - \frac{40000}{x^2}$ dollars per thousand. Total sales are \$38,000 when 1000 watches are sold. What is revenue for 4000 watches?

Marginal revenue
Rate Revenue is changing per item sold.

$$\int \frac{dR}{dx} = \int 60000 - \frac{40000}{x^2}\ x^{-2}$$

$$R(x) = 60000x + \frac{40000x^{-1}}{+1}$$

$$R(x) = 60000x + \frac{40000}{x} + C$$

$$38000 = 60000(1) + \frac{40000}{1} + C$$
$$-100,000$$

$$-62,000 = C$$

$$\boxed{R(x) = 60,000x + \frac{40000}{x} - 62,000}$$

$$\boxed{R(x) = 60{,}000x + \frac{40000}{x} - 62{,}000}$$

(b) Suppose $C(x) = 2000x^2 + \frac{40000}{x} + 20{,}000$

How many should be produced + sold to maximize profit.

$P(x) = R - C$

$$P(x) = \left(60000x + \cancel{\frac{40000}{x}} - 62{,}000\right) - \left(2000x^2 + \cancel{\frac{40000}{x}} + 20000\right)$$

$P(x) = -2000x^2 + 60000x - 82{,}000$

$P'(x) = -4000x + 60{,}000 = 0$

$60{,}000 = 4000x$

$15 = x$

$[0, \infty)$

$\lim_{x \to \infty} -2000x^2 + \cancel{60000x - 82{,}000} = -\infty$

0	-82,000
15	368,000

$\boxed{15{,}000 \text{ watches}}$

Optimization

1) $f'(x) = 0$ critical pts

2) Interval + Test crit pts + end pts. for max/min