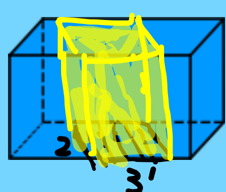
March 26, 2024

FLUID FORCE

Fluid – substance conforms to its container

Vol · ρ

$$F = \rho \cdot h \cdot \underset{l \cdot w}{A}$$



$$F = 62.4 \frac{lb}{\cancel{ft^3}} \cdot 13.5\,\cancel{ft} \cdot 2\,\cancel{ft} \cdot 3\,\cancel{ft}$$

$$= 5054.4 \text{ lb.}$$

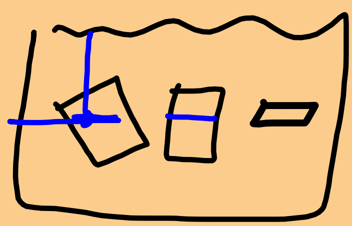
$$\text{Pressure} = \frac{F}{A} = \frac{5054.4}{2 \cdot 3}$$

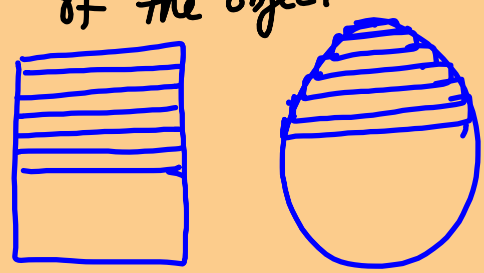
$$= 842.4 \frac{lb}{ft^2}$$

$$842.4 \cdot \frac{lb}{\cancel{ft^2}} \cdot \frac{1\,\cancel{ft^2}}{144 in^2}$$

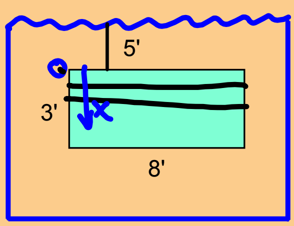
$$= 5.85 \text{ psi}$$

Pascal's Principle – pressure is the same at any depth regardless of the position of the object





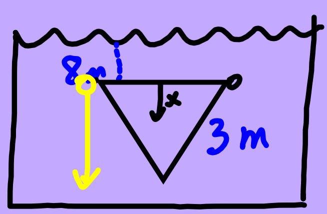
$$F = \rho h A$$



$$\int_a^b \rho \, h(x) \, l(x) \cdot dx$$

$$\int_0^3 62.4(x+5) \cdot 8 \, dx$$

$$= 9734.4 \text{ lb.}$$

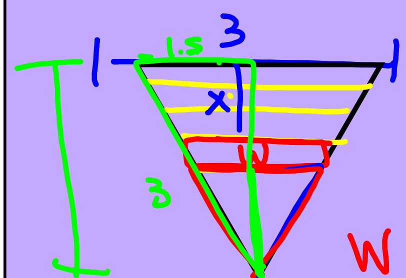


Equilateral Δ

$$\int_a^b \rho\, h(x)\cdot l(x)\, dx$$

$$= \int_0^{\frac{3}{2}\sqrt{3}} 9810(x+8)\cdot \frac{3\left(\frac{3}{2}\sqrt{3}-x\right)}{\frac{3}{2}\sqrt{3}}\, dx$$

$$\approx \boxed{339{,}230\ \text{N}}$$



$x^2 + 1.5^2 = 3^2$

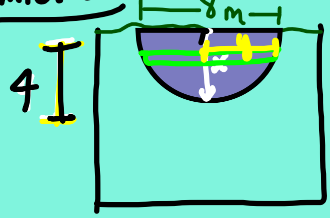
$x = 2.6$

$\frac{3}{2}\sqrt{3}$

$$\frac{W}{3} = \frac{\frac{3}{2}\sqrt{3} - x}{\frac{3}{2}\sqrt{3}}$$

$$W = \frac{3\left(\frac{3}{2}\sqrt{3} - x\right)}{\frac{3}{2}\sqrt{3}}$$

Semicircle



$$\int_a^b \rho \cdot h(x) \cdot l(x)\, dx$$

$$x^2 + y^2 = r^2$$

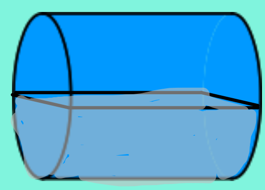
$$x^2 + y^2 = 4^2$$

$$\sqrt{y^2} = \sqrt{16 - x^2}$$

$$y = \pm\sqrt{16 - x^2}$$

$$\int_0^4 9810 \cdot x \cdot 2\sqrt{16 - x^2}\, dx$$

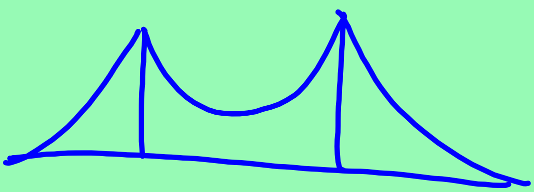
$$= 418{,}560 \text{ N}$$



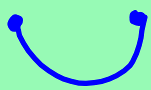
HYPERBOLIC FUNCTIONS

- combinations of e^x + e^{-x}
- properties of trig functions
- connected through complex numbers

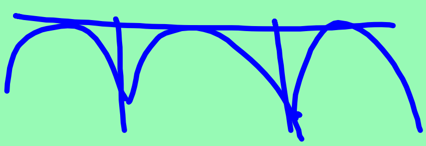
$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$



Catenary



$$\boxed{y = a\cosh\left(\frac{x}{a}\right) + c}$$



$$\sinh(\ln 3) = \frac{e^{\ln 3} - e^{\ln 3^{-1}}}{2}$$

$$= \frac{3 - \frac{1}{3}}{2} = \frac{\frac{8}{3}}{2} = \frac{8}{3}\cdot\frac{1}{2} = \frac{4}{3}$$

Derivatives

$\frac{d}{dx}\sinh x = \cosh x$ $*\frac{d}{dx}\cosh x = \sinh x$

$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$ $\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$

$*\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$ $\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$

$\cosh^2 x - \sinh^2 x = 1$

$f(x) = \coth x \cdot e^{\operatorname{csch} x^3}$ Find $f'(x)$

$$f'(x) = \underbrace{\coth x} \cdot \underbrace{e^{\operatorname{csch} x^3} \cdot -\operatorname{csch} x^3 \coth x^3 \cdot 3x^2}$$

$$+ e^{\operatorname{csch} x^3} \cdot -\operatorname{csch}^2 x$$

$$\int \sinh^7 x \cosh x \, dx$$

$u = \sinh x$

$du = \cosh x \, dx$

$\frac{du}{\cosh x} = dx$

$$\int u^7 \cdot \cancel{\cosh x} \cdot \frac{du}{\cancel{\cosh x}}$$

$$= \frac{u^8}{8} + C$$

$$= \boxed{\frac{\sinh^8 x}{8} + C}$$