$2+4i \quad 3-7i$

POLAR COORDINATES & COMPLEX NUMBERS

$(3+4i)+(2-5i)=5-i$

$i=i$

$i^2=-1$

$i^3=-i$

$i^4=1$

$i=\sqrt{-1}$

$(7+2i)(1+5i) \leftarrow$ FOIL

$=7+35i+2i-10\cancel{i^2}$

$=\boxed{-3+37i}$

$$\frac{4+2i}{3-4i}\cdot\frac{(3+4i)}{(3+4i)}=\frac{12+16i+6i-8\cancel{i^2}}{9+16\cancel{i^2}}=\frac{4+22i}{25}$$

FL

$(7+2i)^8$

$\sqrt[3]{x^3}=\sqrt[3]{-8}$

$x=-2$

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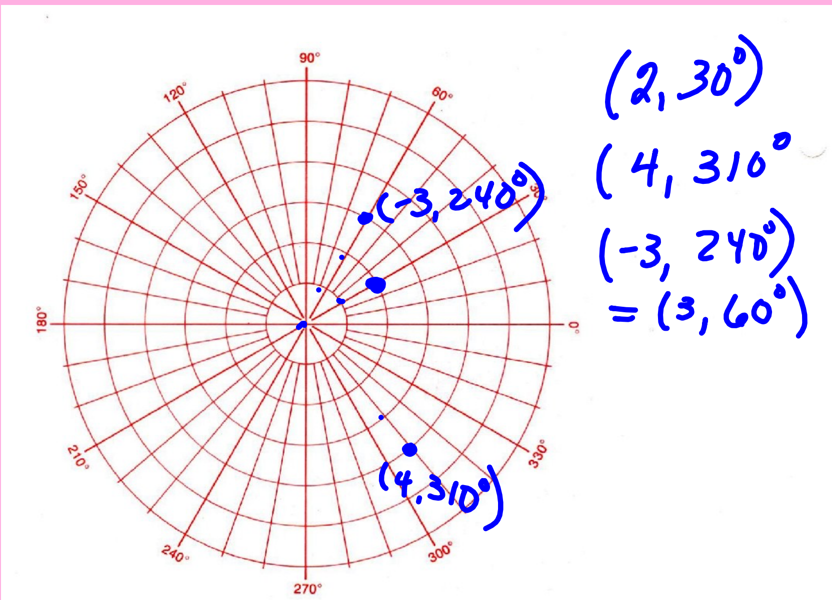


(2, 30°)

(4, 310°

(-3, 240°)

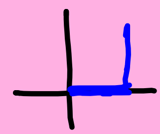
= (3, 60°)

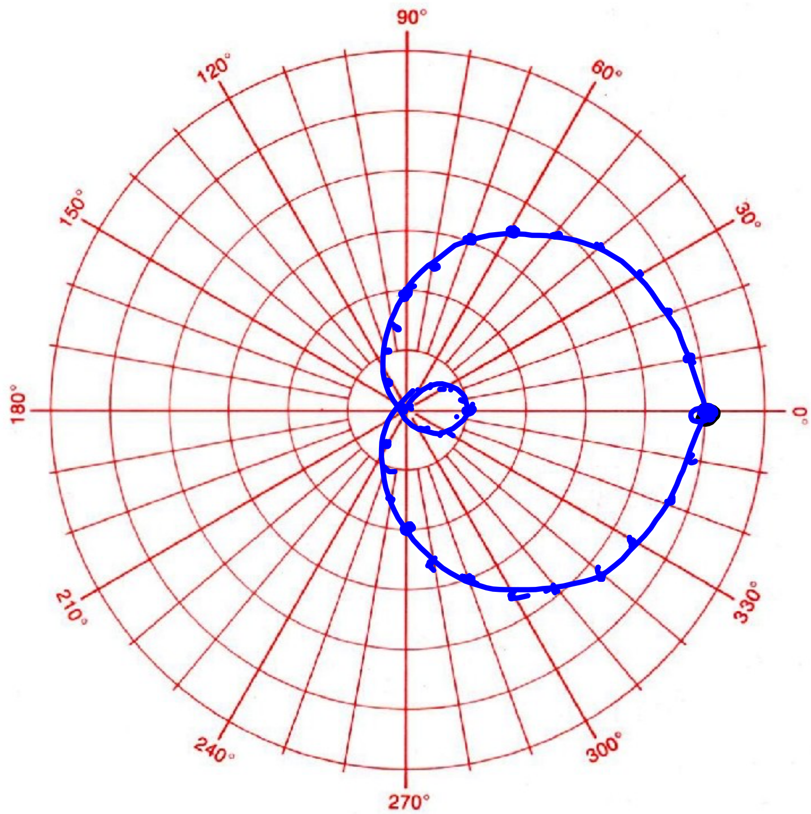
Polar Coordinates

(r, θ)

Rectangular Coord.

(x, y)





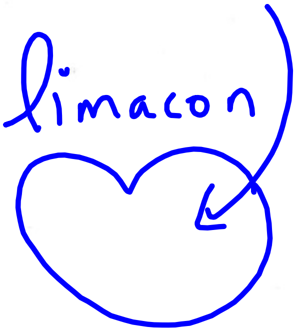
$y = 2x+3$

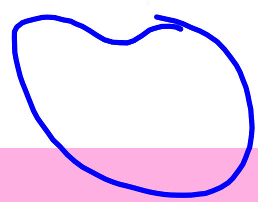
$r = 2+3\cos\theta$

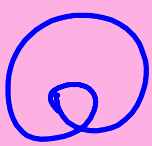
θ	r
$0°$	
$10°$	
$20°$	
$30°$	
$40°$	

cardioid

limacon







$\frac{a}{b} < 1$

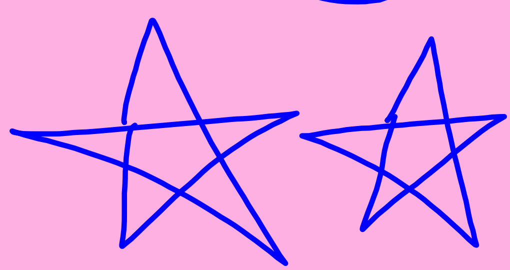
$2+3\cos\theta$

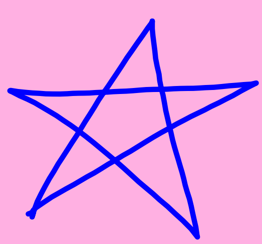
$r = a \pm b\sin\theta$

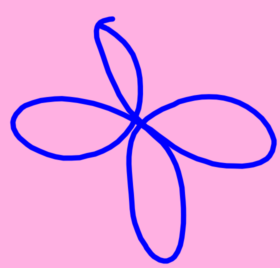
$r = a \pm b\cos\theta$

$\frac{a}{b} = 1$

$3+3\cos\theta$







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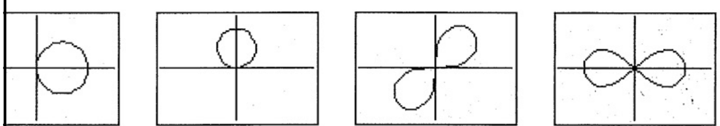
Summary of Polar Graphs The following chart summarizes some of the more common polar graphs and forms of their equations. (In addition to circles, lemniscates, and roses just presented, we include *limaçons.* Cardioids are a special case of limaçons, where $|a/b| \geq 1$.)

Circles and Lemniscates

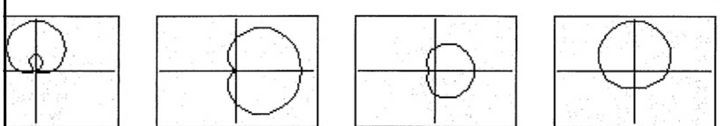
Circles | Lemniscates



$= a \cos \theta$ | $r = a \sin \theta$ | $r^2 = a^2 \sin 2\theta$ | $r^2 = a^2 \cos 2\theta$

Limaçons

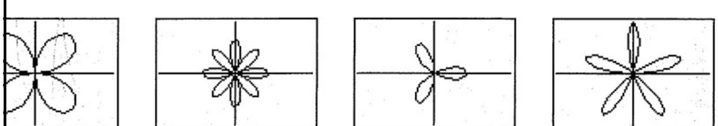
$r = a \pm b \sin \theta$ or $r = a \pm b \cos \theta$



$\frac{a}{b} < 1$ | $\frac{a}{b} = 1$ | $1 < \frac{a}{b} < 2$ | $\frac{a}{b} \geq 2$

Rose Curves

$2n$ petals if n is even, $n \geq 2$ | n petals if n is odd



$n = 2$, $= a \sin n\theta$ | $n = 4$, $r = a \cos n\theta$ | $n = 3$, $r = a \cos n\theta$ | $n = 5$, $r = a \sin n\theta$

Converting between Equation Forms Sometimes an equation given in polar form is easier to graph in rectangular (Cartesian) form. To convert a polar equation to a rectangular equation, we use the following relationships, which were introduced in Section 8.2. See triangle *POQ* in Figure 36.

Converting Coordinates

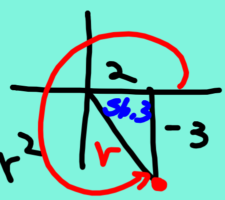
Rectangular	Polar
(x, y)	(r, θ)

Convert to polar coord

$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$(2, -3)$



θ

$$2^2 + (-3)^2 = r^2$$

$$4 + 9 = r^2$$

$$\sqrt{13} = \sqrt{r^2}$$

$$\sqrt{13} = r$$

$$\tan\theta = \frac{-3}{2}$$

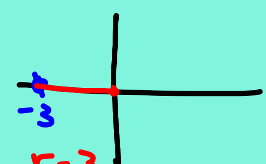
$$\theta = \tan^{-1}(3/2)$$

$$\theta = 56.3^\circ$$

$$\theta = 303.7^\circ$$

$$\boxed{(\sqrt{13}, 303.7^\circ)}$$

$(-3, 0)$



$r = 3$

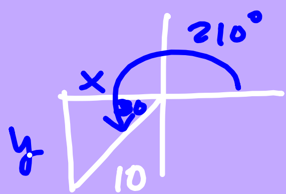
$\theta = 180^\circ$

$$\boxed{(3, 180^\circ)}$$

Polar $\longrightarrow$ Rectangular

$(10, 210^\circ)$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$



$\cos\theta = \frac{x}{r}$ $\quad \sin\theta = \frac{y}{r}$

$r\cos\theta = x$ $\quad r\sin\theta = y$

$x = 10\cos 210^\circ$

$x = 10\left(-\frac{\sqrt{3}}{2}\right) = -5\sqrt{3}$

$y = 10\sin 210^\circ$

$= 10\left(-\frac{1}{2}\right) = -5$

$$\boxed{(-5\sqrt{3}, -5)}$$

Complex Numbers

Rectangular Form

$x + yi$

$-2 + 3i$

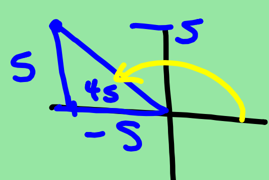
Polar Form (Trigonometric Form)

$r\cos\theta + r\sin\theta\, i$

$$\boxed{r(\cos\theta + i\sin\theta)}$$

Convert to polar form.

$\overset{x}{-5} + \overset{y}{5}i$



$25 + 25 = r^2$

$\sqrt{\underset{25\cdot 2}{50}} = \sqrt{r^2}$

$5\sqrt{2} = r$

$\tan\theta = \frac{5}{-5}$

$\tan\theta = -1$

$\theta = 135^\circ$

$$\boxed{5\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)}$$

Convert to rectangular form.

$2(\cos 300^\circ + i\sin 300^\circ)$

$2\left(+\frac{1}{2} + -i\frac{\sqrt{3}}{2}\right)$

$= \boxed{1 - i\sqrt{3}}$

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