Operations in Polar Form

$$2(\cos 30^\circ + i\sin 30^\circ)\cdot 5(\cos 70^\circ + i\sin 70^\circ)$$

$$10(\cos 30^\circ \cos 70^\circ + i\cos 30^\circ \sin 70^\circ + i\sin 30^\circ \cos 70^\circ \overset{-}{+} \cancel{i^2}\sin 30^\circ \sin 70^\circ)$$

$$10\left(\cos(30^\circ + 70^\circ) + i\sin(30^\circ + 70^\circ)\right)$$

$$10(\cos 100^\circ + i\sin 100^\circ)$$

$$r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2) =$$

$$r_1 r_2\left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$$

$$37(\cos 211^\circ + i\sin 211^\circ)\cdot 4(\cos 348^\circ + i\sin 348^\circ)$$

$$= \boxed{148(\cos 559^\circ + i\sin 559^\circ)}$$

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}\left(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right)$$

Divide & change to rectangular form.

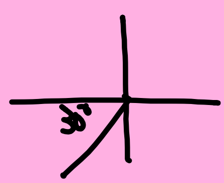
$$\frac{15(\cos 340^\circ + i\sin 340^\circ)}{3(\cos 550^\circ + i\sin 550^\circ)} = 5\left(\cos(340^\circ-550^\circ) + i\sin(340^\circ-550^\circ)\right)$$

$$= 5\left(\cos(-210^\circ) + i\sin(-210^\circ)\right)$$

$$= 5(\cos 210^\circ - i\sin 210^\circ)$$

$$= 5\left(-\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right)$$

$$= -\frac{5\sqrt{3}}{2} + \frac{5}{2}i$$



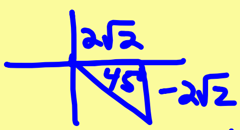
DeMoivre's Theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

$$(2\sqrt{2} - 2i\sqrt{2})^6 = \left[4(\cos 315^\circ + i\sin 315^\circ)\right]^6$$

$$= 4^6(\cos(6\cdot 315^\circ) + i\sin(6\cdot 315))$$



$$(2\sqrt{2})^2 + (2\sqrt{2})^2 = r^2$$
$$8 + 8 = r^2$$
$$16 = r^2$$
$$4 = r$$

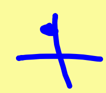
$$\tan\theta = \frac{-2\sqrt{2}}{2\sqrt{2}} = -1$$

$$\theta = 315^\circ$$

$$= 4096(\cos \cancel{1890}\ 90^\circ + i\sin \cancel{1890}\ 90^\circ)$$

$$\frac{1890}{360} = 5.25$$

$$360\cdot 5 = 1800$$



$$= 4096(0 + i\,1)$$

$$= \boxed{4096i}$$

Solve. $x^3+8=0$

$(x^3)^{1/3}=(-8)^{1/3}$

$x=(-8+0i)^{1/3}$

$r=8$

$\theta=180^\circ$

$360^\circ\cdot\frac{1}{3}=120^\circ$

$180^\circ\cdot\frac{1}{3}$

$\left[8(\cos 180^\circ+i\sin 180^\circ)\right]^{1/3} = 8^{1/3}(\cos 60^\circ+i\sin 60^\circ)$

$= 2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$

$= \boxed{1+i\sqrt{3}}$

$\downarrow +360^\circ$

$\left[8(\cos 540^\circ+i\sin 540^\circ)\right]^{1/3} = 2(\cos 180^\circ+i\sin 180^\circ)$

$= 2(-1+0i)$

$= -2$

$\downarrow +360^\circ$

$\left[8(\cos 900^\circ+i\sin 900^\circ)\right]^{1/3} = 2(\cos 300^\circ+i\sin 300^\circ)$

$= 2\left(\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= 1-i\sqrt{3}$

$+120^\circ$

$+120^\circ$

60

1) Isolate the variable. $x^3 = -8$

2) Eliminate the power on the variable by using the 1/n power. $(x^3)^{1/3} = (-8)^{1/3}$

3) Change to polar form. $[8(\cos 180° + i\sin 180°)]^{1/3}$

4) Apply DeMoivre's Theorem. $8^{1/3}(\cos 60° + i\sin 60°)$

5) Get additional answers by taking (1/n)• 360° and add to first answer.

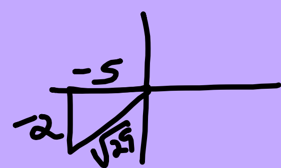
$x^4 - (-5-2i) = 0$

Find the 4th roots of $(-5-2i)$

$(-5-2i)^{1/4}$

$(x^4)^{1/4} = (-5-2i)^{1/4}$

$x = (-5-2i)^{1/4}$



$25 + 4 = r^2$

$\sqrt{29} = \sqrt{r^2}$

$\tan\theta = \frac{+2}{+5}$

$\tan^{-1}(2/5) = 21.8$

$\approx 22^\circ$

$\theta = 180^\circ + 22^\circ$

$= 202^\circ$

$\left[\sqrt{29}(\cos 202^\circ + i\sin 202^\circ)\right]^{1/4}$

$(\sqrt{29})^{1/4} = (29^{1/2})^{1/4} = 29^{1/8}$

$202 \cdot \frac{1}{4} = 50.5$

$360^\circ \cdot \frac{1}{4} = 90^\circ$

1) $29^{1/8}(\cos 50.5^\circ + i\sin 50.5^\circ)$

$+90^\circ$

2) $29^{1/8}(\cos 140.5^\circ + i\sin 140.5^\circ)$

$+90$

3) $29^{1/8} \text{ cis } 230.5^\circ$

$+90$

4) $29^{1/8} \text{ cis } 320.5^\circ$