March 4, 2024

Solving Trig Equations 2

$(2\sin x)^2 = (1-2\cos x)^2 \quad [0°, 360°)$

#6 $\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$

$4\sin^2 x = (1-2\cos x)(1-2\cos x)$

$4\sin^2 x = 1 - 4\cos x + 4\cos^2 x$

$4(1-\cos^2 x) = 1 - 4\cos x + 4\cos^2 x$

$4 - 4\cos^2 x = 1 - 4\cos x + 4\cos^2 x$

$-4 \quad\quad -4$

$0 = 8\cos^2 x - 4\cos x - 3$

$\cos x = \dfrac{4 \pm \sqrt{16 - 4(8)(-3)}}{2(8)}$

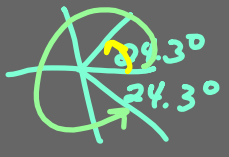
$= \dfrac{4 \pm \sqrt{112}}{16}$

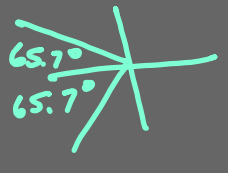
Check:

1) For excluded values with fractions
2) Square both sides

$\cos x = 0.911 \quad\quad \cos x = -0.411$

$\cos^{-1}(0.911\ldots) = 24.3°$





$\cos^{-1}(0.411\ldots) \quad x = 65.7°$

$x = 24.3°, 335.7°, 114.3°, 245.7°$

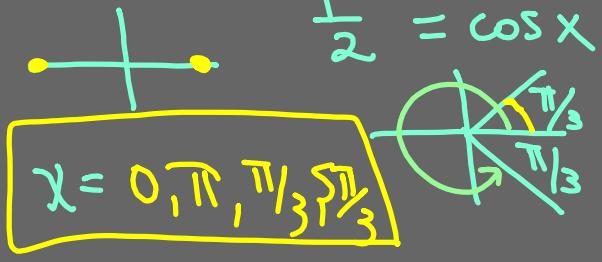
$\sin x - \sin 2x = 0 \quad [0, 2\pi)$

$\sin x - 2\sin x \cos x = 0$

$\sin x (1 - 2\cos x) = 0$

$\sin x = 0 \quad 1 - 2\cos x = 0$

$\frac{1}{2} = \cos x$



$x = 0, \pi, \pi/3, 5\pi/3$

Use identities:

1) Different trig functions

2) Different angles.

$\sin x = \cos\left(\frac{x}{2}\right)$ $[0, 2\pi)$ $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$(\sin x)^2 = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$

$\sin^2 x = \frac{1+\cos x}{2}$

$2\left[1-\cos^2 x = \frac{1+\cos x}{2}\right]$

$2-2\cos^2 x = 1+\cos x$

-2

$0 = 2\cos^2 x + \cos x - 1$

$0 = (2\cos x - 1)(\cos x + 1)$

$\cos x = \frac{1}{2}$ $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

$\sin x = \cos\left(\frac{x}{2}\right)$

$\sin\frac{\pi}{3} = \cos\frac{\pi}{6}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$\sin\frac{5\pi}{3} = \cos\frac{5\pi}{6}$

$-\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

$\sin\pi = \cos\frac{\pi}{2}$

$0 = 0$

$2\cos^2 3\theta + 3\cos 3\theta + 1 = 0$

$(2\cos 3\theta + 1)(\cos 3\theta + 1) = 0$

$\cos 3\theta = -\frac{1}{2}$ $\quad \cos 3\theta = -1$

60° 60°

$3\theta = 120^\circ, 240^\circ, 180^\circ$
$480^\circ, 600^\circ, 540^\circ$
$840^\circ, 960^\circ, 900^\circ$

$\theta = 40^\circ, 80^\circ, 60^\circ$
$160^\circ, 200^\circ, 180^\circ$
$280^\circ, 320^\circ, 300^\circ$

$\theta = [0^\circ, 360^\circ)$
$3\theta = [0^\circ, 1080^\circ)$

If has:
same trig func.
with the
same multiple
angle.

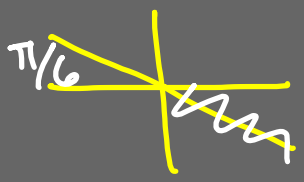
March 4, 2024

$2\tan\left(\frac{x}{2}\right)+\sqrt{3}=-\tan\frac{x}{2}$ $\quad x=[0,2\pi)$

$+\tan\left(\frac{x}{2}\right)$ $\quad \frac{1}{2}x=[0,\pi)$

$$\frac{3\tan\left(\frac{x}{2}\right)}{3}=\frac{-\sqrt{3}}{3}$$

$$\tan\left(\frac{x}{2}\right)=-\frac{\sqrt{3}}{3}$$



$$2\cdot\frac{x}{2}=\frac{5\pi}{6}\cdot 2$$

$$\boxed{x=\frac{5\pi}{3}}$$