Work

$W = \text{Force} \cdot \text{distance}$

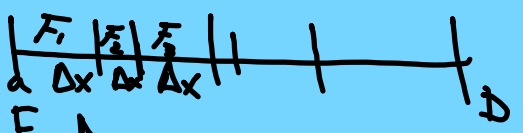
$N \cdot m = Nm$ or Joule

$lbs. \cdot ft = ft \cdot lbs$

Use 20 lb force to move 10 ft.

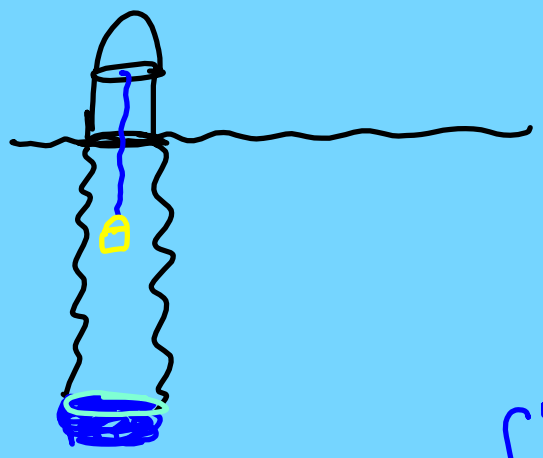
$W = 20\,lb \cdot 10\,ft = 200\,ft \cdot lb.$

Variable Force



$$\lim_{x \to 0} \sum_{x=a}^{b} F_V \Delta x$$

$$W = \int_a^b F(x)\,dx$$



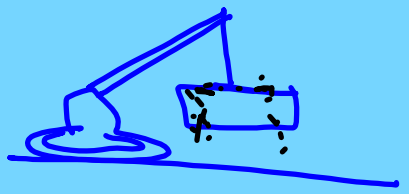
40 ft. deep

When full, bucket weighs 30 lb.

Loses $\frac{1}{4}$ lb for each foot it is raised.

How much work is done to raise the bucket

$$\int_0^{40} \left(30 - \frac{1}{4}x\right) dx = 1000 \text{ ft·lb}$$



Crane - bucket of sand = 500 lb

Raise from ground to 50 ft in 10 sec

Cable weighs 5 lb/ft. $5 \cdot 50 = 250$

Sand spills out 10 lb/sec

How much work to raise the bucket 13.5 ft?

$$W = \int_0^{13.5} \left[\underbrace{(250 - 5x)}_{\text{cable}} + (500 - 2x) \right] dx$$

$\frac{\text{lb}}{\text{ft}}$

$$\frac{10 \text{ lb}}{\text{sec}} \cdot \frac{1 \text{ sec}}{5 \text{ ft}} = 2 \frac{\text{lb}}{\text{ft}}$$

$\approx$ 9487 ft·lb

Springs

Hooke's Law

$F(x) = k \cdot x$

- k: Spring constant
- x: # of units stretched or compressed beyond natural length

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A spring with natural length of 1 m requires a force of 8 N to stretch it 3 m.
How much work to stretch it from 2 m to 4 m?

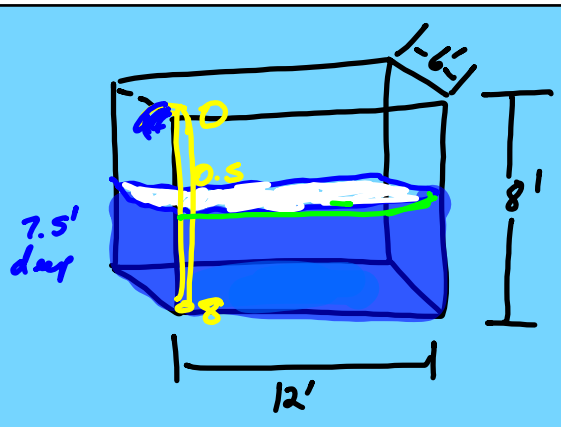
$F = kx$

$8 = k \cdot 3$

$\frac{8}{3} = k$

$W = \int_1^3 \frac{8}{3} x \, dx = \frac{32}{3}$ J

↑ Change from natural length.



Weight density of water:

$\rho = 62.4 \frac{lb}{ft^3}$ OR $9810 \frac{N}{m^3}$

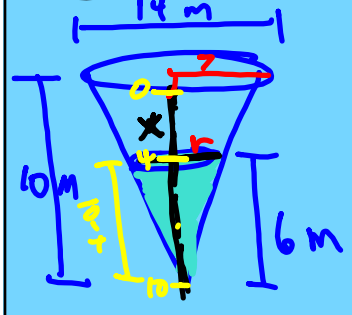
$l \cdot w \cdot h$

$$\int_a^b \rho \cdot A(x) \cdot \text{depth} \cdot dx$$

Weight density — Area of Surface — Depth of water

$$62.4 \int_{0.5}^{8} (12 \cdot 6)\, x \, dx$$

$$\approx \boxed{143{,}208 \text{ ft} \cdot \text{lb}}$$



How much work to pump water out of tank?

$$\int \rho \cdot A(x) \cdot \text{depth} \cdot dx$$

$$9810 \int_4^{10} \pi \left(\frac{7}{10}(10-x)\right)^2 \cdot x \, dx$$

$$\approx 5{,}980{,}123.4 \text{ J}$$

$\frac{7}{r} = \frac{10}{10-x}$

$\frac{7(10-x)}{10} = \frac{10r}{10}$

$\frac{7}{10}(10-x) = r$