

45

$$\sum_{i=2}^8 \frac{2}{i}$$

$$\sum 2^i$$

$$\sum 4i + 3$$

$$= \frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \frac{2}{6} + \frac{2}{7} + \frac{2}{8}$$

$$1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3} + \frac{2}{7} + \frac{1}{4}$$

$$\frac{420}{420} + \frac{280}{420} + \frac{210}{420} + \frac{168}{420} + \frac{140}{420} + \frac{120}{420} + \frac{105}{420} = \frac{1443}{420}$$

$$\frac{481}{140}$$

50

$$\sum_{i=8}^{35} (-3 - 4i)$$

$$\begin{aligned} -3 - 140 &= -143 \\ -3 - 4(35) & \end{aligned}$$

$$\begin{aligned} -3 - 4(8) &= -35 \\ -32 & \end{aligned}$$

$$35 - 8 + 1 = 28$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{28}{2} (-35 + -143)$$

$$= 14 (-178)$$

$$= -2492$$

2/ $\frac{\text{Feb}}{20}$ $\frac{\text{Mar}}{25}$ $\frac{\text{Apr}}{30}$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$\begin{aligned} S_n &= \frac{11}{2} (20 + 70) \\ &= \frac{11}{2} (90) \\ &= 495 \end{aligned}$$

$$a_n = a_1 + d(n-1)$$

$$\begin{aligned} a_n &= 20 + 5(11-1) \\ &= 20 + 50 \\ &= 70 \end{aligned}$$

GEOMETRIC SEQUENCES - multiply by the same value

3, 12, 48, 192,

Common ratio = r

$$r = \frac{a_2}{a_1}$$

81, 54, 36, 24, . . .

$$r = \frac{54}{81} = \frac{6}{9} = \frac{2}{3}$$

5, -15, 45, -135,

$$r = \frac{-15}{5} = -3$$

2, 6, 18, 54,

$r = 3$

2 · 3 2 · 9 2 · 27

2 · 3¹ 2 · 3² 2 · 3³

$$a_n = a_1 \cdot r^{n-1}$$

150, -60, 24,

Find a_8 .

$$r = \frac{-60}{150} = -\frac{2}{5}$$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_8 = 150 \cdot \left(-\frac{2}{5}\right)^{8-1}$$

$$= -\frac{768}{3125}$$

Population of Zeno is decreasing by 8% per year. The population is currently 2100. What will the population be in 7 years?

$$\underbrace{2100, 1932}_{\neq 0.92} \quad r = 100\% - 8\% = 92\% \\ r = 0.92$$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_8 = 2100 \cdot (0.92)^{8-1} \\ \approx 1171 \text{ people}$$

Seneca is growing at 3% per year.

$$100\% + 3\% = 103\% \\ r = 1.03$$

Geometric Series

(312)

$$\begin{array}{r}
 1 S_4 = 2 + 10 + 50 + 250 + 1250 \\
 -5 S_4 = \quad \quad 10 \quad 50 \quad 250 \quad 1250
 \end{array}$$

$$-4 S_4 = 2$$

$$\begin{array}{r}
 -4 S_4 = -1248 \\
 \hline
 -4 \\
 S_4 = 312
 \end{array}$$

$$-1250$$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

Know last term

$$S_n = \frac{a_1 - a_1 \cdot r^{n-1} \cdot r}{1 - r}$$

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r}$$

Know # of terms

Find S_n .

$$6 + 24 + 96 + \dots + 6,291,456.$$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r} = \frac{6 - 6291456 \cdot 4}{1 - 4} = \underline{8388606}$$

$$\sum_{k=2}^9 7 \cdot 3^{k-2}$$

$r=3$ (indicated by a green arrow from the base 3 to the text)
 $a_1 = 7 \cdot 3^{2-2} = 7 \cdot 3^0 = 7 \cdot 1 = 7$
 $r=i$ (indicated by a blue arrow from the text to the base i in i^8)

$$S_n = \frac{a_1 - a_i \cdot r^n}{1 - r} =$$

$n = 9 - 2 + 1 = 8$

$$= \frac{7 - 7 \cdot 3^8}{1 - 3} = \boxed{22960}$$

Infinite Geometric Series

$$4 + 12 + 36 + 108 + \dots = \infty$$

$r=3$

Diverges
(heads off to ∞)
 $r > 1$
(# getting larger)

$$4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$$

$$6 \quad 7 \quad 7.5 \quad 7.75 \quad 7.875 \quad 7.9375 = 8$$

$r = \frac{1}{2}$

Converges
(homes in on a #)

$$r = \frac{2}{4} = \frac{1}{2} \quad S = \frac{a_1}{1-r} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$$

$0 < r < 1$
(# in list getting smaller)

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1-r} = \frac{a_1 - \cancel{a_1 \cdot r^n}}{1-r} = \frac{a_1}{1-r}$$

$$S = \frac{a_1}{1-r}$$