TECHNIQUES OF INTEGRATION

Integration by Parts

$$\int (f\cdot g)' = \int (f\cdot g') + \int (g\cdot f')$$

$$\int x^2 e^x \, dx$$

$$\int f\cdot g' \, dx$$

$$fg = \int f\cdot g' + \int g\cdot f'$$

$$fg - \int g\cdot f' = \int f\cdot g'$$

$$uv - \int v\cdot du = \int u\cdot dv$$

$$\boxed{\int u\cdot dv = uv - \int v\,du}$$

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$$\int u\,dv = uv - \int v\,du$$

$$\int x\sec^2 x\,dx \qquad \begin{array}{ll} u = x & dv = \sec^2 x\,dx \\ du = 1\,dx & v = \tan x \end{array}$$

$$= x\tan x - \int \tan x\,dx$$

$$= x\tan x - \int \frac{\sin x}{\cos x}\,dx \qquad \begin{array}{l} u = \cos x \\ du = -\sin x\,dx \end{array}$$

$$= x\tan x - \int \frac{\cancel{\sin x}}{u}\cdot\frac{du}{-\cancel{\sin x}}$$

$$= x\tan x + \int \frac{1}{u}\,du$$

$$= x\tan x + \ln|u| + C$$

$$= x\tan x + \ln|\cos x| + C$$

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$$\int \ln x \, dx$$

$u = \ln x \quad \int dv = \int dx$

$du = \frac{1}{x} dx \quad v = x$

$$= x \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$$

$$= \boxed{x \ln x - x + C}$$

$$\int x^2 \cdot e^{2x}\,dx \qquad u = x^2 \quad du = 2x\,dx \qquad dv = e^{2x}dx \quad v = \tfrac{1}{2}e^{2x}$$

$$= \frac{1}{2}x^2e^{2x} - \int x e^{2x}\,dx \qquad u = x \quad du = dx \qquad dv = e^{2x} \quad v = \frac{1}{2}e^{2x}$$

$$= \frac{1}{2}x^2e^{2x} + \left[-\frac{1}{2}xe^{2x} + \int \frac{1}{2}e^{2x}\,dx\right]$$

$$= \boxed{\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C}$$

$$\int e^{2x}\,dx \qquad u = 2x \quad du = 2dx$$

$$\int e^u \cdot \frac{du}{2}$$

$$\frac{1}{2}e^u + C$$

$$\frac{1}{2}e^{2x} + C$$

$$\int \sin(8x)\,dx$$

$$-\frac{1}{8}\cos(8x) + C$$

$$\int e^x \cos x \, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x dx \qquad v = \sin x$

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x \qquad v = -\cos x$

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$$

$$\int e^x \cos x \, dx = e^x \sin x + \left[+ e^x \cos x \, dx \mp \int + e^x \cos x \, dx \right.$$

$$\frac{\cancel{2} \int e^x \cos x \, dx}{\cancel{2}} = \boxed{\frac{e^x \sin x + e^x \cos x}{2} + C}$$