

PARTIAL FRACTIONS

$$\left[\frac{15x-14}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2} \right]$$

$(x-2)$ above A and $(x-1)$ above B
 $(x-1)(x-2)$ below the denominator

$$15x-14 = Ax-2A + Bx-B$$

$$15 = A+B$$

$$-14 = -2A-B$$

$$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ -14 \end{bmatrix} = \begin{bmatrix} -1 \\ 16 \end{bmatrix}$$

$$\int \frac{-1}{x-1} dx + \int \frac{16}{x-2} dx$$

$u=x-1$
 $du=dx$

$$\int \frac{-1}{u} du$$

$$= -\ln|x-1| + 16 \ln|x-2| + C$$

$$= \ln|x-2|^{16} - \ln|x-1|$$

$$= \ln \frac{|x-2|^{16}}{|x-1|} + C$$

$$\int \frac{1-x^2}{(4x^2+1)(x^2+4)} dx = \frac{Ax+B}{4x^2+1} + \frac{Cx+D}{x^2+4}$$

$$1-x^2 = (Ax+B)(x^2+4) + (Cx+D)(4x^2+1)$$

$$1-x^2 = Ax^3 + 4Ax + Bx^2 + 4B + 4Cx^3 + Cx + 4Dx^2 + D$$

$$\begin{aligned} 0 &= A & 0 &+ 4C & 0 \\ -1 &= & B & & + 4D \\ 0 &= 4A & & + C & \\ 1 &= & 4B & & + D \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 4 \\ 4 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1/3 \\ 0 \\ -1/3 \end{bmatrix}$$

$$\int \frac{1/3}{4x^2+1} dx + \int \frac{-1/3}{x^2+4} dx$$

$$\frac{1}{3} \int \frac{1}{u^2+1} \frac{du}{2} \Big|_{u=2x} \Big|_{du=2dx} + \frac{-1/3}{3 \cdot 4} \int \frac{1}{\frac{x^2}{4} + 1} dx \quad \begin{aligned} u &= \frac{x}{2} \\ du &= \frac{1}{2} dx \end{aligned}$$

$$\frac{1}{6} \tan^{-1} u$$

$$\frac{1}{6} \tan^{-1}(2x)$$

$$-\frac{1}{12} \int \frac{1}{u^2+1} 2 \cdot du$$

$$= -\frac{1}{6} \tan^{-1} u + C$$

$$= \boxed{\frac{1}{6} \tan^{-1}(2x) - \frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right) + C}$$