CALCULUS SEMESTER 2 REVIEW

List:

A derivative represents...
Integration represents

Give:

$W = \int \rho \, A(x) \cdot \text{depth} \, dx$

$\text{Fluid Force} = \int \rho \, l(x) \, h(x) \, dx$

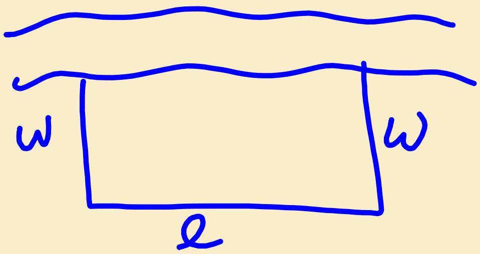
Know:

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

Optimization

$lw = 500$
$(0, \infty)$



$$A = lw$$

$$2w + l = 500$$

$$l = \underline{500 - 2w}$$

$$A = (500 - 2w)\,w$$

$$* \quad A = 500w - 2w^2$$

$$A' = 500 - 4w = 0$$

$$500 = 4w$$

$$125 = w$$

$$(0, 250)$$

$$\lim_{w \to 0} 500w - 2w^2 = 0$$

$$\lim_{w \to 250} 500w - 2w^2 = 0$$

$$A(125) = 500(125) - 2(125)^2$$

$$= \underline{\text{Big\#}}$$

$$l = 500 - 2(125)$$

$$= 250$$

$$125' \times 250'$$

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Hyperbolic Functions

$\frac{d}{dx} \cosh x = \sinh x$

$\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$

$f(x) = \sinh(e^{3x}) \operatorname{sech}(x^3)$

$f'(x) = \underline{\sinh(e^{3x})} \cdot \underline{-\operatorname{sech} x^3 \tanh x^3 \cdot 3x^2} + \operatorname{sech}(x^3) \cosh(e^{3x}) \cdot e^{3x} \cdot 3$

$\int x \tanh^5(3x^2) \operatorname{sech}^2(3x^2)\, dx$

$u = \tanh(3x^2)$

$\frac{du}{dx} = \operatorname{sech}^2(3x^2) \cdot 6x$

$\int \cancel{x} \cdot u^5 \cdot \cancel{\operatorname{sech}^2(3x^2)} \cdot \frac{du}{6\cancel{x} \cancel{\operatorname{sech}^2(3x^2)}}$

$\frac{1}{6} \int u^5 \, du$

$\frac{1}{6} \frac{u^6}{6} + C$

$\frac{1}{36} \tanh^6(3x^2) + C$

Integration

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc^2 x \, dx = -\cot x + C$

$\int \sec x \tan dx = \sec x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

$\int e^x \, dx = e^x + C$

$\int a^x \, dx = \frac{1}{\ln a} \cdot a^x + C$

$\int \frac{1}{x} \, dx = \ln|x| + C$

$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$

$\int \frac{1}{x^2+1} \, dx = \tan^{-1} x + C$

$\int \frac{1}{|x|\sqrt{x^2-1}} \, dx = \sec^{-1} x + C$

Methods:

1) Power Rule
2) U-Subst.
3) Inv Trig Func.
4) Integ by parts
5) Trig Integrals

$\int \sin 4x \cos 8x \, dx$ – Sum & prod. identity

$\int \sin^3 x \cos^4 x \, dx$ – split off $\sin x$
– use $\sin^2 x + \cos^2 x = 1$
– U-sub

$\int \sin^2 x \cos^4 x \, dx$
– Rewrite as $(\ ^2)^-$
– Use double angle Identities

6) Trig Substitution
7) Partial Fractions

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$$\int x(4+2x)^8\,dx$$

$u = 4+2x \Rightarrow \frac{u-4}{2} = \frac{2x}{2}$

$du = 2\,dx$

$\frac{u}{2} - 2 = x$

$$\int x \cdot u^8 \cdot \frac{du}{2}$$

$$\frac{1}{2}\int \left(\frac{u}{2} - 2\right)u^8\,du$$

$$\frac{1}{2}\int \left(\frac{u^9}{2} - 2u^8\right)du$$

$$\frac{1}{2}\left[\frac{u^{10}}{20} - \frac{2u^9}{9}\right] + C$$

$$\frac{(4+2x)^{10}}{40} - \frac{(4+2x)^9}{9} + C$$

$$\frac{d}{dx}\int_2^{x^3} \frac{(5-t^3)^7}{\sin t}\,dt =$$

$$= \frac{\left(5-(x^3)^3\right)^7}{\sin x^3} \cdot 3x^2$$

$$= \frac{3x^2(5-x^9)^7}{\sin x^3}$$