SEMESTER REVIEW

Factoring

2 terms

$a^2 - b^2$ Conjugates

$a^2 + b^2$ — not factorable

$\left.\begin{array}{l} a^3 + b^3 \\ a^3 - b^3 \end{array}\right]$ Square – multiply – Square

3 terms

UNFOIL

4 terms

Grouping

$$\frac{x^2 - 4}{x^3 - 8} \div \frac{3x^3 + 6x^2 - 4x - 8}{x^2 - 6x - 8}$$

$$\frac{x^2 - 4}{x^3 - 8} \cdot \frac{x^2 - 6x + 8}{(3x^3 + 6x^2 - 4x - 8)}$$

$$2^3$$

$$\frac{(x+2)(x-2)}{(x-2)(x^2+2x+4)} \cdot \frac{(x-4)(x-2)}{3x^2(x+2) + 4(x+2)} = \frac{(x-4)(x-2)}{(x^2+2x+4)(3x^2+4)}$$

$$(x+2)(3x^2+4)$$

Synthetic Division — only possible if ÷ by $x+\#$ or $x-\#$

$$\frac{4x^3 - 2x + 8}{x + 3}$$

$$x^5 + 2x^4 - 3x^2 + 2x - 5 = 0$$
Future Excitement!

$$-3 \begin{array}{|cccc} 4 & 0 & -2 & 8 \\ & -12 & 36 & -102 \\ \hline 4 & -12 & 34 & -94 \end{array}$$

$$= 4x^2 - 12x + 34 - \frac{94}{x+3}$$

$$x^3 \cdot x^5 = x^8$$

$$7^4 \cdot 7^{-6} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

$$(3f^4 g^{-2})^{-3} = 3^{-3} f^{-12} g^6$$
$$= \frac{g^6}{3^3 f^{12}} = \frac{g^6}{27 f^{12}}$$

$$\frac{a^{8-4} b^4 c^{-2+5} d^{1-9}}{a^4 b^{11-5} c^{-8} d^{-1+9}} = \frac{a^4 c^3}{b^6 d^8}$$

$$(2ab)^0 + 2ab^0 = 1 + 2a$$

$$\boxed{1 + 2a}$$

Roots

even

$\sqrt[4]{18a^4b^3c^{10}} \cdot \sqrt[4]{9a^6b^{17}c^{20}}$

$2^4 = 16$
$3^4 = 81$
$4^4 = 256$

$= \sqrt[4]{162a^{10}b^{20}c^{30}}$
$81 \cdot 2$

$= 3a^2b^5|c^7|\sqrt[4]{2a^2c^2}$

Even – Even – Odd
index   power   power $= | \ |$
        inside outside

— must check original problem!

$\sqrt[6]{f^2g^2} \cdot \sqrt[4]{f g^3}$

$= \sqrt[12]{f^4g^2} \cdot \sqrt[12]{f^3g^9}$

$= \sqrt[12]{f^7 g^{11}}$

Evaluate.

$4^{5/2} = \sqrt[2]{4^5} = 2^5 = 32$

$125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \boxed{\dfrac{1}{5}}$

Solving radical equations

$2\sqrt{x-4} + 1 = 9$

$2\sqrt{x-4} = 8$

$\left(\sqrt{x-4}\right)^2 = (4)^2$

$x - 4 = 16$

$x = 20$

1) Isolate the root

2) Square both side

Check. $2\sqrt{\underset{16}{20-4}} + 1 = 9$

$2 \cdot 4 + 1 = 9$

$8 + 1 = 9 \checkmark$

$\sqrt{2x+3} - \sqrt{x-2} = 2$

$1^2 + 2^2 = 3^2$

$\left(\sqrt{2x+3}\right)^2 = \left(2 + \sqrt{x-2}\right)^2$

FOIL!

$2x + 3 = \left(2 + \sqrt{x-2}\right)\left(2 + \sqrt{x-2}\right)$

$2x + 3 = 4 + 2\sqrt{x-2} + 2\sqrt{x-2} + x - 2$ ← Clean Up

$2x + 3 = 2 + x + 4\sqrt{x-2}$ ← Isolate & Square

$\underset{-x\ -2}{2x+3} = \underset{-2\ -x}{2 + x}$

FOIL! $(x+1)^2 = \left(4\sqrt{x-2}\right)^2$

$(x+1)(x+1) = 16(x-2)$

$x^2 + 2x + 1 = 16x - 32$

$x^2 - 14x + 33 = 0$

$(x-11)(x-3) = 0$

$\boxed{x = 11 \quad x = 3}$

Check!

Check:

$\sqrt{25} - \sqrt{9} = 2$

$5 - 3 = 2 \checkmark$

$\sqrt{9} - \sqrt{1} = 2$

$3 - 1 = 2 \checkmark$

$f(x) = 2x - 7 \quad g(x) = x^2 - 4$

$(g \circ f)(3)$

$f(3) = 2(3) - 7 = -1$

$g(-1) = (-1)^2 - 4$

$= \boxed{-3}$

$(f \circ g)(x) = 2(x^2 - 4) - 7$

$= 2x^2 - 8 - 7$

$= 2x^2 - 15$

Find $g^{-1}(x)$.

1) Switch $x + y$.
2) Solve for y.

$y = x^2 - 4$

$x = y^2 - 4$

$\sqrt{x + 4} = \sqrt{y^2}$

$\pm\sqrt{x + 4} = g^{-1}$

22(u)

$$\frac{2x}{x^2-4} + \frac{1}{x^2+3x+2}$$

Make common denom —

1) Factor the denom.

2) Make common denom

$$\frac{(x+1)\, 2x}{(x+1)(x+2)(x-2)} + \frac{1\,(x-2)}{(x+1)(x+2)(x-2)}$$

$$\frac{2x^2+2x+x-2}{(x+2)(x-2)(x+1)} = \frac{\overset{(2x-1)(x+2)}{2x^2+3x-2}}{(x+2)(x-2)(x+1)} = \frac{(2x-1)(x+2)}{(x+2)(x-2)(x+1)}$$