TRIG INTEGRALS

Use sum + product identity

$$\int \sin 4x \cos 3x \, dx$$

$$\frac{1}{2}\int \left[\sin(4x+3x) + \sin(4x-3x)\right] dx$$

$$\frac{1}{2}\int \left(\sin 7x + \sin x\right) dx$$

$$= \frac{1}{2}\left[-\frac{1}{7}\cos 7x - \cos x\right] + C$$

$$= -\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + C$$

$$\int \sin^5 x \, dx$$

$$\int \sin^4 x \cdot \sin x \, dx$$

$$\int (\sin^2 x)^2 \cdot \sin x \, dx$$

$$\int (1-\cos^2 x)^2 \sin x \, dx$$

$$\int (1-u^2)^2 \cdot \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}}$$

$$-\int (1-u^2)(1-u^2) \, du$$

$$-\int (1-2u^2+u^4) \, du$$

$$-\left[u - \frac{2u^3}{3} + \frac{u^5}{5}\right] + C$$

$$\boxed{-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C}$$

<u>If one term has an odd power</u>

1) split off $\sin x$ or $\cos x$ from the odd power.
2) Rewrite the remaining term as $(\sin^2 x)^p$ or $(\cos^2 x)^p$.
3) Use $\sin^2 x + \cos^2 x = 1$ to rewrite squared term.
4) u-sub, simplify, & integrate

$u = \cos x$

$du = -\sin x \, dx$

$\frac{du}{-\sin x} = dx$

$$\int \sin^6(2x)\ \cos^3(2x)\, dx$$

$$\int \sin^6(2x)\, \cos^2(2x) \cdot \underline{\cos(2x)}\, dx$$

$$\int \sin^6(2x)\left(1 - \sin^2(2x)\right) \cdot \cos 2x\, dx$$

$u = \sin 2x$

$$\int \left(\sin^6(2x) - \sin^8(2x)\right) \cos 2x\, dx$$

$du = \cos(2x) \cdot 2\, dx$

$$\int \left(u^6 - u^8\right) \cancel{\cos 2x} \cdot \frac{du}{2\cancel{\cos 2x}}$$

$$\frac{1}{2}\left[\frac{u^7}{7} - \frac{u^9}{9}\right] + C$$

$$\frac{\sin^7(2x)}{14} - \frac{\sin^9(2x)}{18} + C$$

$\cos 2x = 1 - 2\sin^2 x$

$\frac{1}{2} \cdot 2\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos 2x = 2\cos^2 x - 1$

$\frac{1 + \cos 2x}{2} = \frac{2\cos^2 x}{2}$

$\frac{1}{2}(1 + \cos 4x) = \cos^2 2x$

$\cos 2(2x)$

Both powers even

1) Rewrite all as $(\sin^2 x)^p$ or $(\cos^2 x)^p$

2) Use double angle identity to rewrite $\sin^2 x$ + $\cos^2 x$

3) Foil + integrate each term separately.

$$\int \cos^4 x \, dx$$

$$\int (\cos^2 x)^2 \, dx$$

$$\int \left[\frac{1}{2}(1 + \cos(2x))\right]^2 dx$$

$$\frac{1}{4} \int (1 + \cos 2x)(1 + \cos 2x) \, dx$$

$$\frac{1}{4} \int \left(1 + 2\cos(2x) + \cos^2(2x)\right) dx$$

$\frac{1}{\cancel{2}} \cdot \cancel{2} \sin(2x)$

$$\frac{1}{4}\left[x + \sin 2x + \int \frac{1}{2}(1 + \cos 4x)\, dx\right.$$

$u = 4x$

$$\int \cos 4x \, dx$$

$$\frac{1}{4}\sin 4x + C$$

$$\frac{1}{4}\left[x + \sin 2x + \frac{1}{2}x + \frac{1}{2} \cdot \frac{1}{4}\sin 4x\right] + C$$

$$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + C$$

$$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

12/ $\int \sin^2 x \cos^4 x \, dx$

$$\int \sin^2 x \cdot (\cos^2 x)^2 \, dx$$

$$\int \tfrac{1}{2}(1-\cos 2x) \cdot \left[\tfrac{1}{2}(1+\cos 2x)\right]^2 dx$$

$$\frac{1}{8}\int \left[(1-\cos 2x)(1+\cos 2x)(1+\cos 2x)\right] dx$$

$$\frac{1}{8}\int (1-\cos^2(2x))(1+\cos 2x) \, dx$$

$$\frac{1}{8}\int \sin^2(2x)(1+\cos 2x) \, dx$$

$$\frac{1}{8}\left[\int \sin^2(2x)\, dx + \int \sin^2(2x)\cos(2x)\, dx\right.$$

↑ double angle identity

u-sub