TRIG INTEGRALS

$$\int \sin 4x \cos 3x \, dx$$

Use sum + product identity

$$\frac{1}{2}\int \left[\sin(4x+3x) + \sin(4x-3x)\right] dx$$

$$\frac{1}{2}\int (\sin 7x + \sin x)\, dx$$

$$= \frac{1}{2}\left[-\frac{1}{7}\cos 7x - \cos x\right] + C$$

$$= -\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + C$$

$$\int \sin^5 x \, dx$$

$$\int \sin^4 x \cdot \sin x \, dx$$

$$\int (\sin^2 x)^2 \cdot \sin x \, dx$$

$$\int (1-\cos^2 x)^2 \sin x \, dx$$

$$\int (1-u^2)^2 \cdot \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}}$$

$$-\int (1-u^2)(1-u^2) \, du$$

$$-\int (1-2u^2+u^4) \, du$$

$$-\left[u - \frac{2u^3}{3} + \frac{u^5}{5}\right] + C$$

$$\boxed{-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C}$$

<u>If one term has an odd power</u>

1) split off $\sin x$ or $\cos x$ from the odd power.
2) Rewrite the remaining term as $(\sin^2 x)^p$ or $(\cos^2 x)^p$.
3) Use $\sin^2 x + \cos^2 x = 1$ to rewrite squared term.
4) u-sub, simplify, & integrate

$u = \cos x$

$du = -\sin x \, dx$

$\frac{du}{-\sin x} = dx$

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$$\int \sin^6(2x)\ \cos^3(2x)\,dx$$

$$\int \sin^6(2x)\cos^2(2x)\cdot \underline{\cos(2x)}\,dx$$

$$\int \sin^6(2x)\left(1-\sin^2(2x)\right)\cdot\cos 2x\,dx$$

$$u=\sin 2x$$

$$\frac{du}{dx}=\cos(2x)\cdot 2$$

$$\int\left(\sin^6(2x)-\sin^8(2x)\right)\cos 2x\,dx$$

$$\int\left(u^6-u^8\right)\cancel{\cos 2x}\cdot\frac{du}{2\cancel{\cos}2x}$$

$$\frac{1}{2}\left[\frac{u^7}{7}-\frac{u^9}{9}\right]+C$$

$$\frac{\sin^7(2x)}{14}-\frac{\sin^9(2x)}{18}+C$$

$\cos 2x = 1 - 2\sin^2 x$

$\frac{1}{2} \cdot 2\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos 2x = 2\cos^2 x - 1$

$\frac{1 + \cos 2x}{2} = \frac{2\cos^2 x}{2}$

$\frac{1}{2}(1 + \cos \cancel{4x}) = \cos^2 \cancel{x}\,2x$

$\cos 2(2x)$

Both powers even

1) Rewrite all as $(\sin^2 x)^p$ or $(\cos^2 x)^p$
2) Use double angle identity to rewrite $\sin^2 x + \cos^2 x$
3) Foil + integrate each term separately.

$\int \cos^4 x \, dx$

$\int (\cos^2 x)^2 \, dx$

$\int \left[\frac{1}{2}(1 + \cos(2x))\right]^2 dx$

$\frac{1}{4}\int (1 + \cos 2x)(1 + \cos 2x)\, dx$

$\frac{1}{4}\int \left(1 + 2\cos(2x) + \cos^2(2x)\right) dx$

$\frac{1}{\cancel{2}} \cdot \cancel{2} \sin(2x)$

$u = 4x$

$\int \cos 4x \, dx$

$\frac{1}{4}\sin 4x + C$

$\frac{1}{4}\left[x + \sin 2x + \int \frac{1}{2}(1 + \cos 4x)\, dx\right.$

$\frac{1}{4}\left[x + \sin 2x + \frac{1}{2}x + \frac{1}{2} \cdot \frac{1}{4}\sin 4x\right] + C$

$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + C$

$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

12/ $\int \sin^2 x \cos^4 x \, dx$

$$\int \sin^2 x \cdot (\cos^2 x)^2 \, dx$$

$$\int \tfrac{1}{2}(1-\cos 2x) \cdot \left[\tfrac{1}{2}(1+\cos 2x)\right]^2 dx$$

$$\tfrac{1}{8}\int \left[(1-\cos 2x)(1+\cos 2x)(1+\cos 2x)\right] dx$$

$$\tfrac{1}{8}\int (1-\cos^2(2x))(1+\cos 2x)\, dx$$

$$\tfrac{1}{8}\int \sin^2(2x)(1+\cos 2x)\, dx$$

$$\tfrac{1}{8}\left[\int \sin^2(2x)\,dx + \int \sin^2(2x)\cos(2x)\,dx\right.$$

$u = \sin 2x$

$du = \cos(2x) \cdot 2\, dx$

$$\tfrac{1}{8}\left[\tfrac{1}{2}\int (1-\cos(2x))\,dx + \int u^2 \cdot \cancel{\cos 2x} \cdot \frac{du}{2\cancel{\cos 2x}}\right.$$

$$\tfrac{1}{8}\left[\tfrac{1}{2}\left(x - \tfrac{1}{2}\sin(2x)\right) + \tfrac{1}{2}\cdot\tfrac{u^3}{3}\right] + C$$

$$\tfrac{1}{8}\left[\tfrac{1}{2}x - \tfrac{1}{4}\sin 2x + \tfrac{1}{6}\sin^3(2x)\right] + C$$

$$\tfrac{1}{16}x - \tfrac{1}{32}\sin 2x + \tfrac{1}{48}\sin^3(2x) + C$$

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Trig Substitution

$a^2 - x^2$

$x = a\sin\theta$

$a^2 + x^2$

$x = a\tan\theta$

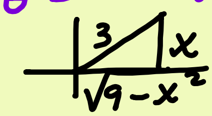
$x^2 - a^2$

$x = a\sec\theta$

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx \qquad x = 3\sin\theta \qquad dx = 3\cos\theta\, d\theta$$

$\frac{y}{r}$ $\quad \frac{x}{3} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$



$$\int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} \cdot 3\cos\theta\, d\theta$$

$$\frac{27}{3}\int \frac{\sin^2\theta\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$$

$$9\int \frac{\sin^2\theta\cdot\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta$$

$$9\int \frac{\sin^2\theta\,\cancel{\cos\theta}}{\cancel{\cos\theta}}$$

$$9\int \frac{1}{2}\left(1-\cos 2\theta\right)d\theta$$

$$\frac{9}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

$$= \frac{9}{2}\theta - \frac{9}{4}\sin 2\theta + C$$

$$= \frac{9}{2}\theta - \frac{9}{4}\sin(2\theta) + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{4}\cdot 2\sin\theta\underset{\frac{x}{r}}{\cos\theta} + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{2}\right) - \frac{\cancel{9}}{2}\left(\frac{x}{\cancel{3}}\right)\left(\frac{\sqrt{9-x^2}}{\cancel{3}}\right) + C$$

$$= \boxed{\frac{9}{2}\sin^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}x\sqrt{9-x^2} + C}$$

$$\int \frac{\cos\theta}{\sin^2\theta}\,d\theta \qquad u=\sin\theta,\quad du=\cos\theta\,d\theta$$

$$\int \frac{\cancel{\cos\theta}}{u^2}\cdot\frac{du}{\cancel{\cos\theta}} = \int u^{-2}\,du = \frac{u^{-1}}{-1}+C = \frac{-1}{u}+C$$

$$= \boxed{\frac{-1}{\sin\theta}+C}$$

$$\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta \qquad \int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$$

$$\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

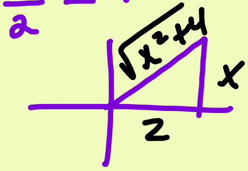
$$\int \frac{1}{\sec^2\theta}\,d\theta = \int \cos^2\theta\,d\theta = \text{see a)}$$

$$\int \tan^2\theta\,d\theta = \int(\sec^2\theta-1)\,d\theta = \tan\theta-\theta+C$$

#7) $1+\tan^2 x=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx \qquad x = 2\tan\theta \qquad \frac{y}{x}\ \ \frac{x}{2} = \tan\theta$$
$$dx = 2\sec^2\theta\,d\theta$$



$$\int \frac{1}{\underset{2}{\cancel{4}}\tan^2\theta\sqrt{4\tan^2\theta+4}}\cdot\cancel{2}\sec^2\theta\,d\theta$$

$$\frac{1}{2\cdot 2}\int \frac{\sec^2\theta\,d\theta}{\tan^2\theta\sqrt{\tan^2\theta+1}}$$

$$\frac{1}{4}\int \frac{\sec^{\cancel{2}}\theta}{\tan^2\theta\,\cancel{\sqrt{\sec^2\theta}}\ \cancel{\sec\theta}}$$

$$\frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$\frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$\frac{1}{4}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta} \qquad u = \sin\theta, \quad du = \cos\theta\,d\theta$$

$$\frac{1}{4}\int \frac{\cancel{\cos\theta}}{u^2}\cdot\frac{du}{\cancel{\cos\theta}}$$

$$\frac{1}{4}\int u^{-2}\,du$$

$$\frac{1}{4}\,\frac{u^{-1}}{-1} + C$$

$$\frac{-1}{4\sin\theta} + C$$

$$= \frac{-1}{4\sin\theta} + C \quad y/r$$

$$= \frac{-1}{4\,\frac{x}{\sqrt{x^2+4}}}$$

$$= \boxed{-\frac{\sqrt{x^2+4}}{4x} + C}$$