

# FRACTALS

1979/1980 - Benoit Mandelbrot

Dynamical Systems - anything that moves or changes in time

- \* Weather prediction
- \* Stock market
- \* Chemical reactions

Iteration

$$f(x) = x^2 + C$$

$$f(x) = x^2 + 0+0i$$

$$\text{Seed value } x_0 = 0$$

$$f(0) = 0^2 + 0+0i = 0$$

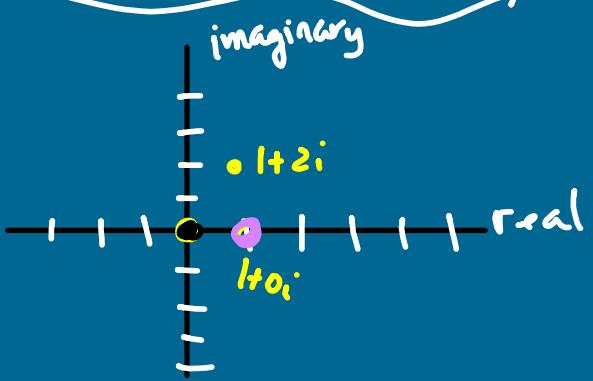
$$f(0) = 0$$

fixed point = iteration stays on same value

Orbit = the list of #'s that result from iteration

1, 2, 5, 26, ...

goes to  $\infty$ .



$$f(x) = x^2 + (1+0i)$$

$$f(0) = 0^2 + 1 = 1$$

$$f(1) = 1^2 + 1 = 2$$

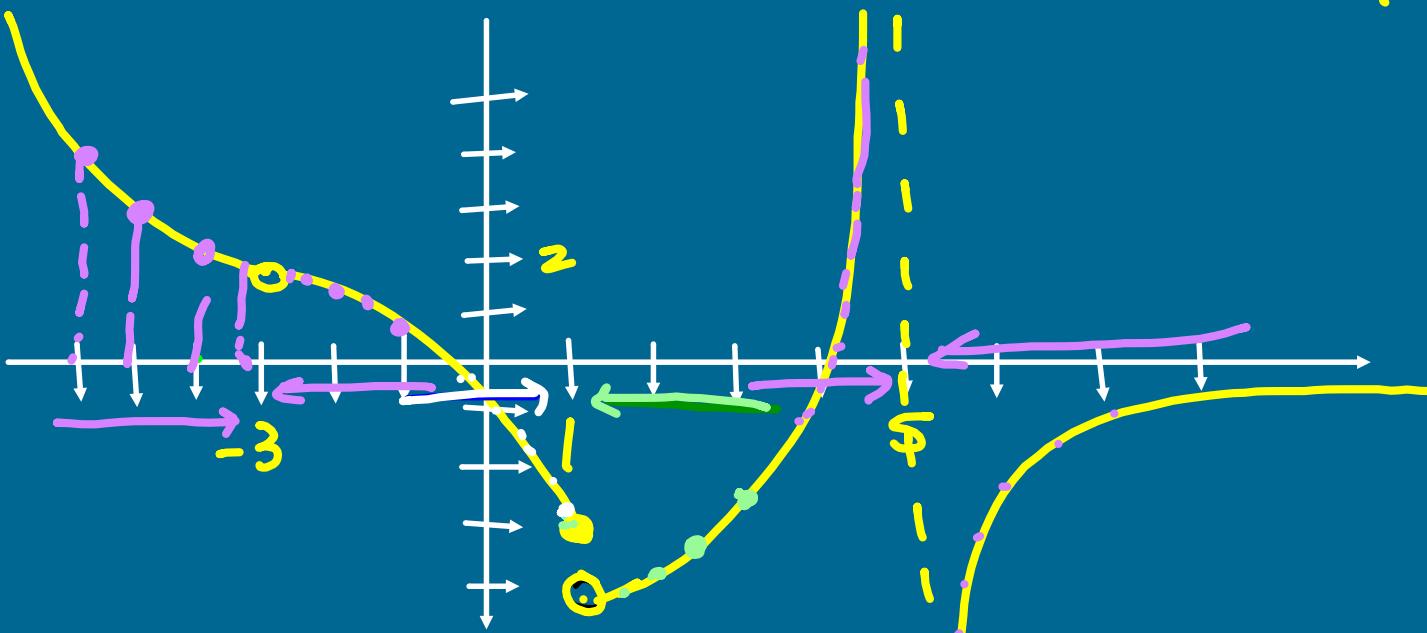
$$f(2) = 2^2 + 1 = 5$$

$$f(5) = 5^2 + 1 = 26$$

$$f(26) = 26^2 + 1 = \text{Big}$$

# INTRO TO CALCULUS

- LIMITS  
DERIVATIVES  
INTEGRALS



$$\lim_{x \rightarrow -3^-} f(x) = 2$$

$$\lim_{x \rightarrow -3^+} f(x) = 2$$

$$\lim_{x \rightarrow 3} f(x) = 2$$

$f(-3) = \text{undef.}$

$$\lim_{x \rightarrow 1^-} f(x) = -3$$

$$\lim_{x \rightarrow 1^+} f(x) = -4$$

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

DNE  
Does Not Exist

$$f(1) = -3$$

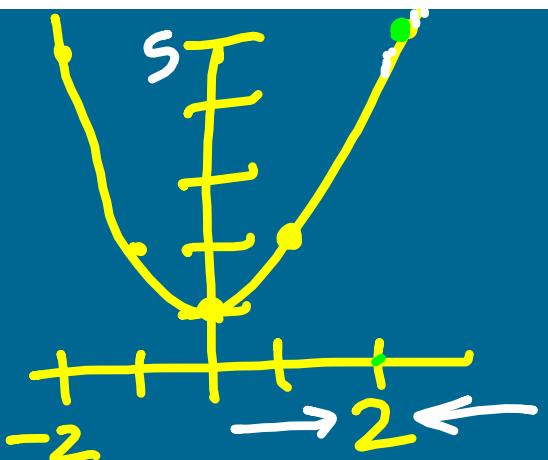
$$\lim_{x \rightarrow 5^-} f(x) = +\infty$$

$$\lim_{x \rightarrow 5^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 5} f(x) = \text{DNE}$$

$$f(5) = \text{undef.}$$

$$\lim_{x \rightarrow 2} \frac{x^2+1}{x^2+1} = 2^2 + 1 = 5$$

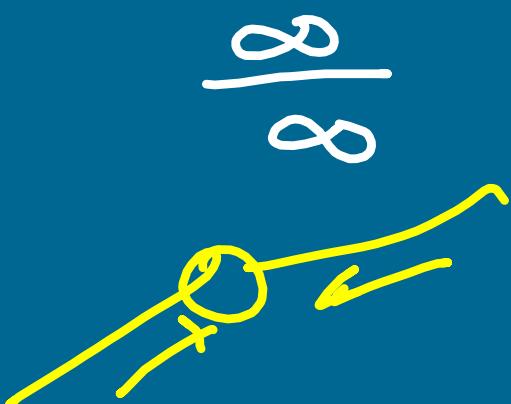


1) Sub # in  
2) If 0, try factoring or conjugates

$$\lim_{x \rightarrow -4} \frac{x^2 - 3x}{\sqrt{x+8}} = \frac{16 + 12}{\sqrt{-4+8}} = \frac{28}{2} = 14$$

$$\lim_{x \rightarrow 3} \frac{x^2 + 4x - 21}{x^2 - 3x} = \frac{9 + 12 - 21}{9 - 9} = \frac{0}{0} \text{ indeterminate}$$

$$\lim_{x \rightarrow 3} \frac{(x+7)(x-3)}{x(x-3)} = \frac{3+7}{3} = \frac{10}{3}$$



$$\lim_{x \rightarrow -5} \frac{x^3 + 125}{x^2 - 25} = \frac{0}{0}$$

$$\lim_{x \rightarrow -5} \frac{(x+5)(x^2 - 5x + 25)}{(x-5)(x+5)} = \frac{25 + 25 + 25}{-10} = \frac{75}{-10} = -\frac{15}{2}$$

$$\lim_{x \rightarrow 64} \frac{\sqrt{x} - 8}{x - 64} \stackrel{\sqrt{x} + 8}{=} \frac{8 - 8}{64 - 64} = \frac{0}{0}$$

$$\lim_{x \rightarrow 64} \frac{x - 64}{(\cancel{x-64})(\sqrt{x} + 8)}$$

$$= \frac{1}{\sqrt{64} + 8} = \frac{1}{16}$$