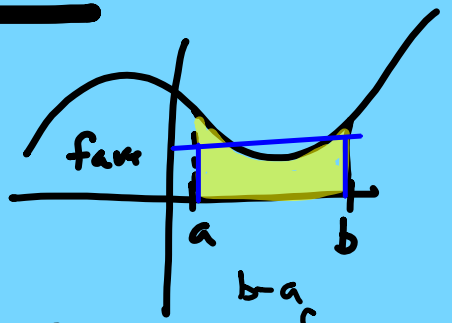
Semester Review

Find average value (f_{ave})

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$



$$f_{ave}\cdot(b-a) = \int f(x)$$

$f(x) = 2x^3+4 \quad [1,3]$

$$f_{ave} = \frac{1}{3-1}\int_1^3 (2x^3+4)\,dx$$

$$= \frac{1}{2}\left[\frac{2x^4}{4^2} + 4x\right]_1^3$$

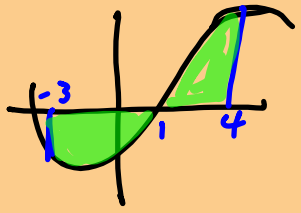
$$= \frac{1}{2}\left[\frac{81}{2} + 12 + \left(-\frac{1}{2} - 4\right)\right]$$

$40 + 8$

$$= \frac{1}{2}[48]$$

$$= 24$$

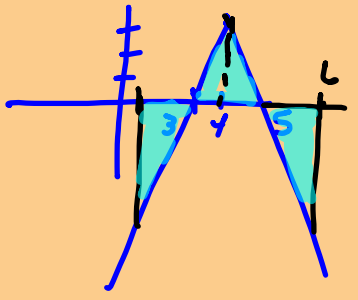
Area & Volume

Area = 

$$-\int_{-3}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx$$

$[1, 6]$

$f(x) = -3|x-4|+3$



$-3(x-4)+3 = -3x+15$

$3(x-4)+3 = 3x-9$

Disk

$\pi\int_a^b (r_o)^2-(r_i)^2\,dx$

▯ is ⊥ to axis of rev.

Shell

$2\pi\int_a^b r(f-g)\,dx$

▯ is ∥ to axis of rev.

Slicing

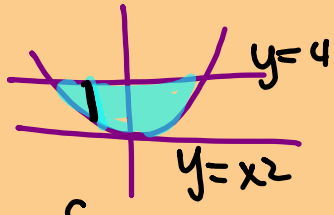
$\int_a^b A(x)\,dx$

Squares $A = s^2$

Isos. Rt Δ $A = \frac{1}{2}s^2$

Equil. Δ $A = \frac{\sqrt{3}}{4}s^2$

Semicircles $A = \frac{1}{2}\pi r^2$



Squares $\int (4-x^2)^2\,dx$

Semicircles $\pi\int \left(\frac{4-x^2}{2}\right)^2 dx$

B(b) $\sqrt{y^2} = \pm\sqrt{x^2-16}$ $y=0$, $x=5$

around y-axis

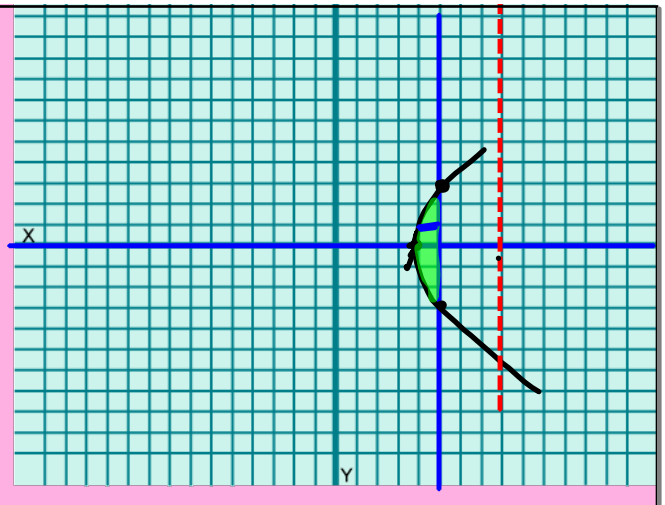
$y = \pm\sqrt{x^2-16}$

4	0
5	3

$y^2 = x^2 - 16$

$\sqrt{y^2+16} = \sqrt{x^2}$

$$\pi \int_0^3 \left[5^2 - \left(\sqrt{y^2+16}\right)^2\right] dy$$



Around $x = 8$

$$\pi \int_0^3 \left(8 - \sqrt{y^2+16}\right)^2 - (8-5)^2 \Big] dy$$

B(b) $\sqrt{y^2} = \pm\sqrt{x^2-16}$ $y=0$ $x=5$

around y-axis

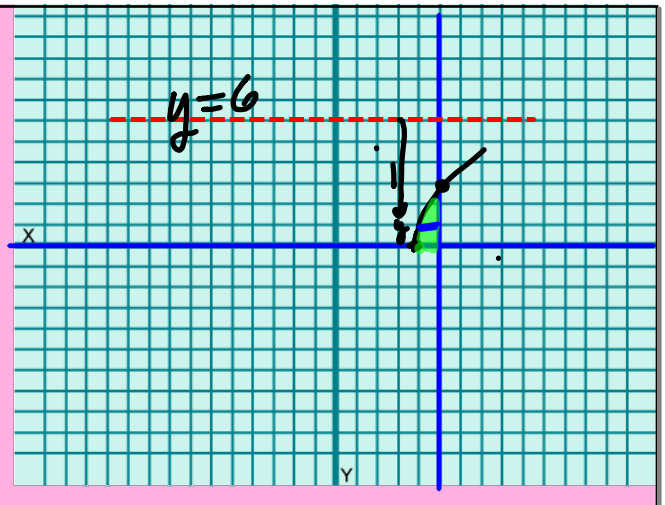
Shell Method $x = \sqrt{y^2+16}$

$y = 6$

r (f - g)

$$2\pi \int_0^3 (6-y)\left(5-\sqrt{y^2+16}\right) dy$$

T - B R - L



Length of Curve

$$\int_a^b \sqrt{1+[f'(x)]^2}\, dx$$

Surface Area

$$\int_a^b f(x)\sqrt{1+[f'(x)]^2}\, dx$$

Find particular solution

$\int \frac{dy}{dx} = \int x^4 + 4x^3 + 2x + 1$ $\quad y = 4$ when $x = -2$

$y =$ _____ $+ C$

Solve for C

$a(t)$

$v(t) = \int a(t)\,dt$

$s(t) = \int v(t)\,dt$

$s(t)$

$v(t) = s'(t)$

$\int a(t) = \int 2t$

$v(t) = t^2 + C$

$20 = 4^2 + C$

$4 = C$

$\boxed{v(t) = t^2 + 4}$

20 ft/sec after 4 (sec)

Work

Springs

$F(x) = Kx$

Natural = 5 ft

Force of = 40 lb moves 8 ft. past natural length

$40 = K \cdot 8$

$5 = K$

How much force to stretch it 7 to 10 ft?

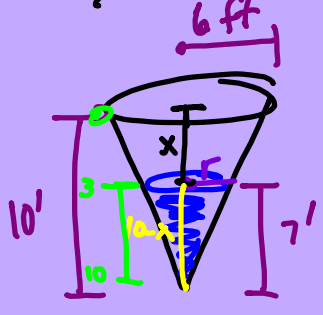
$$\int_2^5 5x\,dx$$

Helicopter = 40 ft

Chain = 5 lb/ft

Bucket of water = 500 lb. losing 3 lb/ft.

$$\int_0^{40} (500 - 3x) + (200 - 5x)$$



$$\frac{r}{6} = \frac{10-x}{10}$$

$$r = \frac{6}{10}(10-x)$$

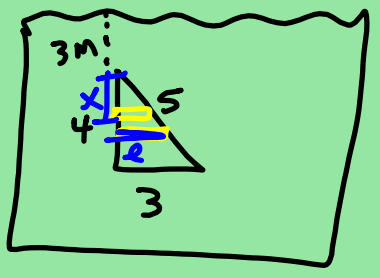
Pump water out of tank.

$$\rho \int_a^b A(x) \cdot \text{depth} \cdot dx$$

$$62.4 \int_3^{10} \pi \left[\frac{6}{10}(10-x)\right]^2 \cdot x\,dx$$

(r^2)

Fluid Force



$$\rho \int \quad l(x) \cdot h(x)\, dx \qquad \text{depth}$$

$$\rho \int \quad (\quad)(x+3)\, dx$$

May 1, 2024