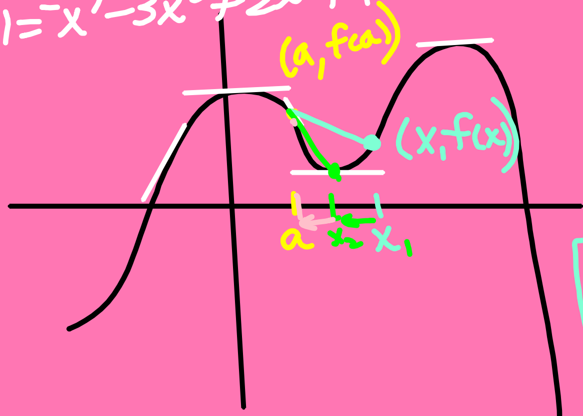
September 17, 2024

DERIVATIVES

– represent the slope of a line tangent to a curve at a given point.

$f(x) = -x^4 - 3x^3 + 2x^2 + 1$



$$m = \frac{f(x) - f(a)}{x - a}$$

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

1st Def. of Deriv.

Let's x get really close to a & become a tangent line

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

Notation

Func	1st Deriv	2nd Deriv
$f(x)$	$f'(x)$	$f''(x)$
$y =$	$y' =$	$y'' =$
$y =$	$\frac{dy}{dx}$	$\frac{d^2}{dx^2}y$

Isaac Newton

Gottfried von Leibniz

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f(x) = x^3 - 2x^2 + x - 1$$

$$\lim_{x \to a} \frac{x^3 - 2x^2 + x - \cancel{1} + (-a^3 + 2a^2 - a + \cancel{1})}{x - a}$$

$$\lim_{x \to a} \frac{(x^3 - a^3)\overset{-2(x^2-a^2)}{(-2x^2 + 2a^2)}(+x - a)}{x - a}$$

$$\lim_{x \to a} \frac{\cancel{(x-a)}(x^2 + ax + a^2) - 2\cancel{(x-a)}(x+a) + \overset{1}{\cancel{x-a}}}{\cancel{x-a}}$$

$$\lim_{x \to a} (x^2 + ax + a^2) - 2(x+a) + 1$$

$$= a^2 + a^2 + a^2 - 2(2a) + 1$$

$$= \boxed{3a^2 - 4a + 1}$$

Find slope at $a = 2$

$M = 3(2)^2 - 4(2) + 1$

$m = 12 - 8 + 1$

$= 5$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{\cancel{x}+h-\cancel{x}}$$

$$\boxed{\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

2nd Def. of Deriv.

$f(x) = \sin x$

$$\cancel{\lim_{x\to a}\frac{\sin x-\sin a}{x-a}}$$

$f(x) = 3x^2 - 2x + 5$

$$\lim_{h\to 0}\frac{3(x+h)^2 - 2(x+h) + 5 + (-3x^2 + 2x - 5)}{h}$$

$3(x^2+2xh+h^2)$

$$\lim_{h\to 0}\frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h + \cancel{5} - \cancel{3x^2} + \cancel{2x} - \cancel{5}}{h}$$

$$\lim_{h\to 0}\frac{6xh + 3h^2 - 2h}{h}$$

$$\lim_{h\to 0}\frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}}$$

$f' = \boxed{6x - 2x^0}$

$3x^2 - 2x^1 + 5x^0$

$$f(x) = 8\sin x$$

$$\lim_{h\to 0} \frac{8\sin(\overset{A+B}{x+h}) - 8\sin x}{h}$$

$$8\lim_{h\to 0} \frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$

$$8\lim_{h\to 0} \frac{\sin x\cos h - \sin x}{h} + \frac{\cos x\sin h}{h}$$

$$8\lim_{h\to 0} \sin x\frac{(\cos h - 1)}{h} + \cos x\cdot\frac{\sin h}{h}$$

$$= 8\left[\cancel{\sin x\cdot 0} + \cos x\cdot 1\right]$$

$$= \boxed{8\cos x}$$

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$f(x) = 4\tan x + 3\csc x$$

$$f'(x) = 4\sec^2 x + -3\csc x \cot x$$

$$\boxed{\text{Power Rule} \quad \frac{d}{dx} x^n = n \cdot x^{n-1}}$$

- all terms in numerator
- all + or −

$$f(x) = 3x^8 - \frac{1}{3x^5} - 7\sqrt[3]{x^2} + 3$$

$$= 3x^8 - \frac{1}{3}x^{-5} - 7x^{2/3} + 3$$

$$f'(x) = 24x^7 + \frac{5}{3}x^{-6} - \frac{14}{3}x^{-1/3}$$

$$= \boxed{24x^7 + \frac{5}{3x^6} - \frac{14}{3x^{1/3}}}$$

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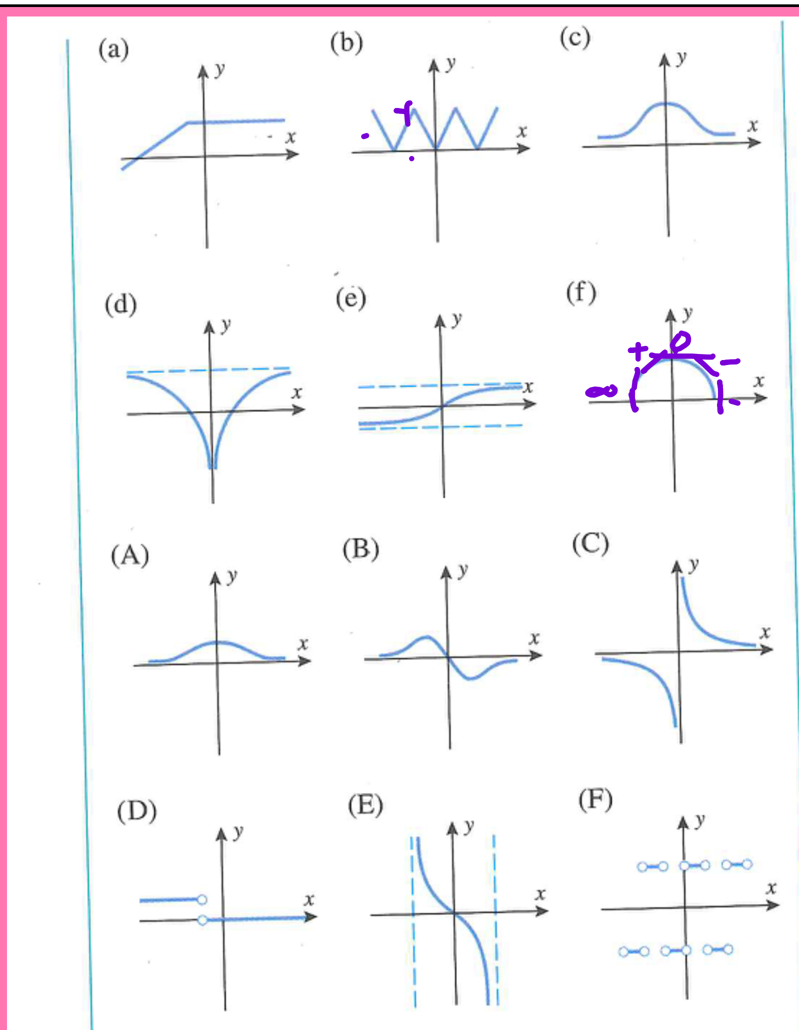

How to find eq. of tangent line.

$f(x) = 2x^2 - 3x + 1$ at $(-1, 6)$

1) Need to fill in $\boxed{y - y_1 = m(x - x_1)}$
2) Know the point.
3) Find slope of tangent line – DERIVATIVE!

$f'(x) = 4x - 3$

x-coord of pt. $f'(-1) = 4(-1) - 3$

$= -7$

$m = -7$

$y - 6 = -7(x + 1)$

$y - 6 = -7x - 7$

$\boxed{y = -7x - 1}$