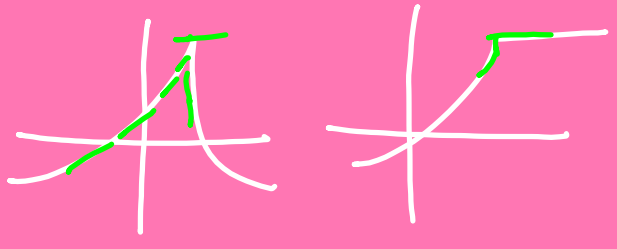
September 27, 2024

DIFFERENTIABILITY



1) must be continuous

2) Differentiable

no sharp pts,
no sudden change in slope
no vertical tangent line

1) $f(a)$ is defined.
2) $\lim_{x \to a} f(x)$ exists.
3) $f(a) = \lim_{x \to a} f(x)$ ☺
4) $f'(a)^- = f'(a)^+$

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 4 \\ 6 - x & \text{if } x < 4 \end{cases}; \quad a = 4$$

1) $f(4) = \sqrt{4} = 2$
2) $\lim_{x \to 4^+} \sqrt{x} = 2$

$\lim_{x \to 4^-} (6 - x) = 2$

$\lim_{x \to 4} f(x) = 2$

3) $f(4) = \lim_{x \to 4} f(x)$

f is continuous ☺

4) $f'(4)^+ = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{4}} = 1/4$

$f'(4)^- = -1$

$f'(4)^- \neq f'(4)^+$

f is not differentiable.

Derivatives Review

A derivative represents. . . .

2 Definitions

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Deriv. of 6 Trig Func

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$f(x) = \tan^6(x^2-4x)\sec(3x^4-5x^8)^9$$

$$f'(x) = \tan^6(x^2-4x)\cdot \sec(3x^4-5x^8)^9 \tan(3x^4-5x^8)^9 \cdot 9(3x^4-5x^8)^8 \cdot (12x^3-40x^7) + \sec(3x^4-5x^8)^9 \cdot 6\tan^5(x^2-4x) \cdot \sec^2(x^2-4x)\cdot(2x-4)$$

18-19 Find $f''(x)$

$f(x) = (x^2-4)^{3/4}$

$f'(x) = \frac{3}{4}(x^2-4)^{-1/4} \cdot 2x$

$= \left(\frac{3}{2}x\right)\cdot(x^2-4)^{-1/4}$

$f''(x) = \frac{3}{2}x \cdot -\frac{1}{4}(x^2-4)^{-5/4} \cdot 2x + (x^2-4)^{-1/2} \cdot \frac{3}{2}$

Write eq. of tangent line.

$y = \sin\left(\frac{x}{2}\right)$ $\quad x = \frac{\pi}{2}$

$\frac{1}{2}x$

Point:

$y = \sin\left(\frac{\pi}{\frac{2}{1}}\right) = \sin \pi/4$

$= \sqrt{2}/2$

$\left(\pi/2, \sqrt{2}/2\right)$

Slope:

$y' = \cos\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$m = \cos\left(\frac{\pi/2}{2}\right)\cdot\frac{1}{2}$

$m = \sqrt{2}/2 \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{2}/2 = \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{2}\right)$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}x - \frac{\sqrt{2}\pi}{8}$

$+ \frac{\sqrt{2}}{2} = \frac{4\sqrt{2}}{8}$

$y = \frac{\sqrt{2}}{4}x - \frac{\sqrt{2}\pi}{8} + \frac{4\sqrt{2}}{8}$

$y = \frac{\sqrt{2}}{4}x - \frac{\sqrt{2}}{8}(\pi - 4)$

1 Differentiability Proof = Notate correctly!

1 or 2 Differential problems